

# Impact of physics problem characteristics on problem difficulty

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# What “non-conceptual” properties can increase problem difficulty?

- Use of potentially misleading/distracting diagrammatic elements
- Inclusion of (potentially misleading) irrelevant or redundant information
- Use of symbols in place of numbers (e.g.,  $m$  or  $\mu$  for *mass*)
- Dependence on or use of unfamiliar or subtle assumptions or terms
  - e.g., *adiabatic, thermal reservoir, quasi-static, reversible, spontaneous*

**Investigating the impact of problem properties on introductory and advanced student responses to introductory thermodynamics conceptual problems**

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# Targeted Concepts (among others)

( $E$  = internal energy;  $Q$  = heat transfer *to* system)

- $E$  is proportional to  $T$  for an ideal gas
- The sign of  $\Delta E$  for an ideal gas is determined by whether the product  $PV$  is increasing or decreasing
- In a reversible isothermal process,  $Q \neq 0$

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**Information provided to students on first page:**

**Survey of Thermodynamic Processes and First and Second Laws  
(STPFaSL-Long)**

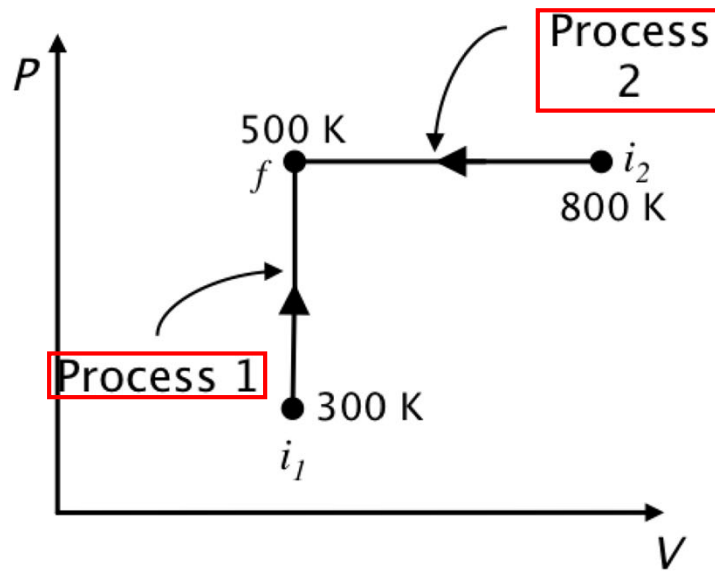
- Please select only one of the four choices, (a)-(d) or True/False for each of the questions.
- All temperatures  $T$  are absolute temperatures.
- All experiments involving a gas as the system are performed with a fixed amount of gas.
- The following equations may be useful for an ideal monatomic gas system where the symbols have the usual meaning: the internal energy  $E_{int} = (3/2)NkT$  and  $PV = NkT$ .
- Thermal reservoirs are significantly larger than the system so that heat transfer between the system and the reservoir does not change the temperature of the reservoir.
- An adiabatic process is one in which there is no heat transfer between a system and its surroundings.
- The process described in questions 60-62 is quasi-static. A quasi-static process passes through a sequence of equilibrium states.

The following abbreviations are used throughout the survey:

- $W$  = work done by the system.
- $Q$  = net heat transfer to the system.

Also,  $Q_1$ ,  $Q_2$  in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes end in the same state shown on the  $PV$  diagram below. Process 1 is a constant volume process starting at 300K at point  $i_1$  and ending at point  $f$  at 500K whereas process 2 is a constant pressure process starting at 800K at point  $i_2$  and ending at point  $f$  at 500K. Answer the following two questions about these experiments:



- (33) Which one of the following statements is true about the change in the internal energy of the gas for **process 1**?
- Internal energy does not change for process 1.
  - Internal energy increases for process 1.
  - Internal energy decreases for process 1.
  - Not enough information.

**Answer: (b); temperature increases so internal energy increases.**

**Correct-response rate; Calculus-based: 76%; Algebra-based: 81%**

- (34) Which one of the following statements is true about the change in the internal energy of the gas for **process 2**?
- Internal energy does not change for process 2.
  - Internal energy increases for process 2.
  - Internal energy decreases for process 2.
  - Not enough information.

**Answer: (c); temperature decreases so internal energy decreases.**

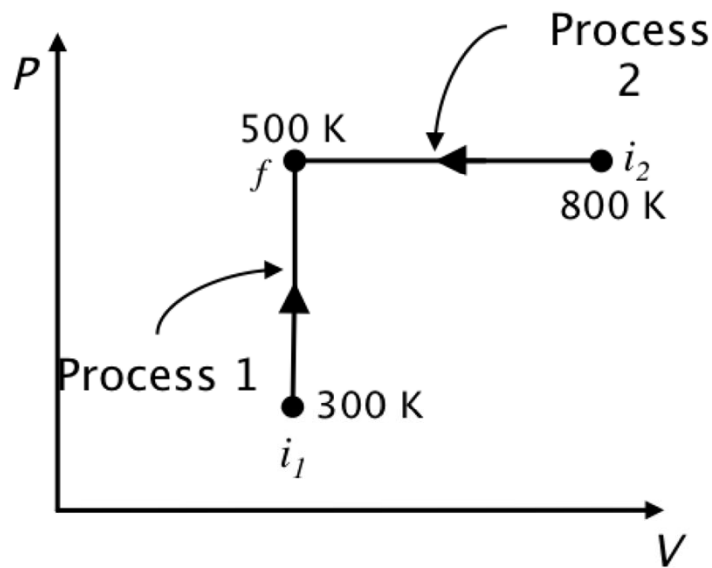
**Correct-response rate; Calculus-based: 58%; Algebra-based: 55%**

**Many more errors on #34 (Process 2) →**

# Why did students make significantly more errors on #34?

**Interviews:** 3 of 6 students who gave incorrect answer on #34 (Process 2) argued “Work done is negative so internal energy will increase”

*Students were excessively distracted by work considerations on Process 2.*



- (34) Which one of the following statements is true about the change in the internal energy of the gas for process 2?
- Internal energy does not change for process 2.
  - Internal energy increases for process 2.
  - Internal energy decreases for process 2.
  - Not enough information.

**Answer: (c); temperature decreases so internal energy decreases.**

**Correct-response rate; Calculus-based: 58%; Algebra-based: 55%**

The use of a horizontal arrow on a  $PV$  diagram (instead of an arrow pointed straight up) lured some students into unproductive considerations regarding *work*.

# Targeted Concepts (among others)

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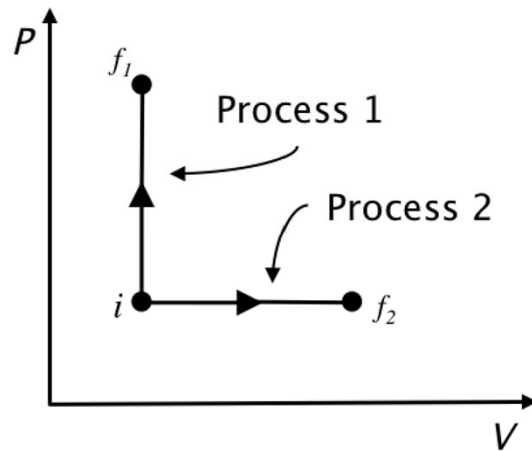
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The following abbreviations are used throughout the survey:

- $W$  = work done by the system.
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Also,  $Q_1$ ,  $Q_2$  in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes start in the same state  $i$  as shown on the  $PV$  diagram below. Process 1 is a constant volume (isochoric) process and process 2 is a constant pressure (isobaric) process. Answer the following six questions about the two processes:



- (44) Which one of the following statements is correct about the change in internal energy of the gas in **process 1**?
- There is no change in the internal energy of the gas in process 1.
  - The internal energy of the gas increases in process 1.
  - The internal energy of the gas decreases in process 1.
  - Not enough information.
- (45) Which one of the following statements is correct about the change in internal energy of the gas in **process 2**?
- There is no change in the internal energy of the gas in process 2.
  - The internal energy of the gas increases in process 2.
  - The internal energy of the gas decreases in process 2.
  - Not enough information.

[N ≈ 500]

**Answer to both #44 and #45: (b);**  $PV$  increases so internal energy increases [ $E_{int} = (3/2) NkT = (3/2) PV$ ].

Correct-response rate on Process 1, Calculus-based: 69%; Algebra-based: 70%

Correct-response rate on Process 2, Calculus-based: 43%; Algebra-based: 37% **← More incorrect answers on Process 2**

**Interviews:** 7 of 9 incorrect responses on Process 2 were justified by saying work done is positive so energy would decrease, thus ignoring the role of heat transfer.

The use of a horizontal arrow on a  $PV$  diagram (instead of an arrow pointed straight up) lured students into unproductive considerations regarding *work*, lowering the correct-response rate by 26-33%.



# Targeted Concepts (among others)

( $E$  = internal energy;  $W$  = work done *by* system;  $Q$  = heat transfer *to* system)

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# Targeted Concepts (among others)

( $E$  = internal energy;  $W$  = work done *by* system;  $Q$  = heat transfer *to* system)

- $E$  is proportional to  $T$  for an ideal gas
- The sign of  $\Delta E$  for an ideal gas undergoing an isochoric or isobaric process is determined by whether pressure or volume are increasing
- In a reversible isothermal process,  $Q \neq 0$

An ideal gas is allowed to undergo an **isothermal expansion**. Answer the following three questions about this process.

$$Q - W = \Delta E = 0 \text{ since } \Delta T = 0, \text{ so } Q = W > 0$$

(60) Which one of the following statements is true about the heat transfer in this process?

- a. There is no net heat transfer between the gas and its environment.
- b. There is net heat transfer to the gas.**
- c. There is net heat transfer away from the gas.
- d. Not enough information.

**Correct-response rate**

Calculus-based: 43%; Algebra-based: 44%

**Q = 0 responses**

Calculus-based: 30%; Algebra-based: 37%

You perform an experiment with an ideal monatomic gas such that it undergoes a **reversible isothermal compression**. Answer questions (4) and (5) below about this experiment.

$$Q - W = \Delta E = 0 \text{ since } \Delta T = 0, \text{ so } Q = W < 0$$

(4) Which one of the following is true about the net heat transfer during the reversible isothermal compression?

- a. There is net heat transfer to the gas.
- b. There is net heat transfer away from the gas.**
- c. There is no net heat transfer to or from the gas.
- d. Not enough information.

**Correct-response rate**

Calculus-based: 30%; Algebra-based: 21%

**Q = 0 responses**

Calculus-based: 50%; Algebra-based: 59%



*Many more Q = 0 responses on “reversible compression” question, justified by “process is isothermal” arguments*

# General Findings

- Horizontal process arrows on *PV* diagrams often triggered unproductive lines of reasoning regarding work
- Certain terms may trigger unproductive lines of reasoning (e.g., *reversible*)

# Symbolic Procedures

**Confusion of symbolic meaning:** Students perform worse on solving problems when symbols are used to represent common physical quantities in equations [Torigoe and Gladding, 2007; 2011)

**Example [Multiple-choice questions; University of Illinois]:**

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance  $d$  from the start at rest is the car traveling 30 m/s?

[93% correct]

Version #2: A car can go from 0 to  $v_1$  in  $t_1$  seconds. At what distance  $d$  from the start at rest is the car traveling  $(v_1/2)$ ?

[57% correct]



Much worse!

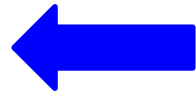
➤ *Our results on “stripped-down” versions are analogous, although differences are smaller*

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{v_1}{t_1}$$

$$v = \frac{v_1}{2}$$



Symbolic version

$$d = ?$$

- A.  $d = v_1 t_1$    B.  $d = \frac{v_1 t_1}{2}$    C.  $d = \frac{v_1 t_1}{4}$    D.  $d = \frac{v_1 t_1}{8}$    E.  $d = \frac{v_1 t_1}{16}$

Numeric version



$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

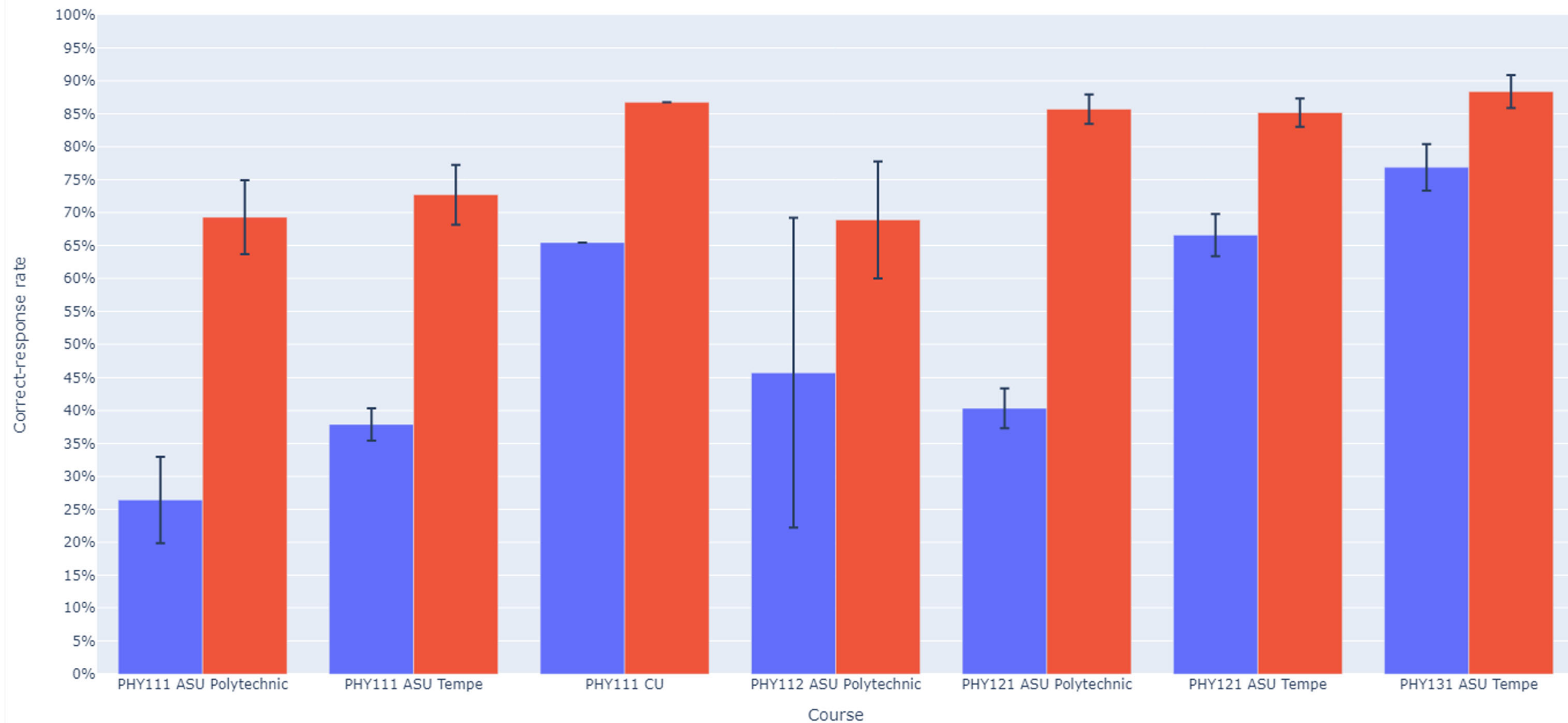
$$d = ?$$

- A.  $d = 30$    B.  $d = 60$    C.  $d = 120$    D.  $d = 240$    E.  $d = 480$

Symbolic version:



Numeric version:



## Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$$0.5y = 2x$$

$$78.4 - y = 8x$$

[Solve for  $x$ ]

**Numeric Version** 61% correct ( $N = 470$ )

## Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$$\begin{array}{l} 0.5y = 2x \\ 78.4 - y = 8x \end{array} \quad [\text{Solve for } x] \quad \text{Numeric Version} \quad 61\% \text{ correct } (N = 470)$$

$$\begin{array}{l} cy = dx \\ a - y = bx \end{array} \quad [\text{Solve for } x] \quad \text{Symbolic Version} \quad 31\% \text{ correct } (N = 372)$$

## Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$$0.5y = 2x$$

$$78.4 - y = 8x$$

[Solve for  $x$ ]

**Numeric Version** 79% correct ( $N = 1205$ )

## Algebra: Simultaneous Equations (calculus-based course, ASU-T)

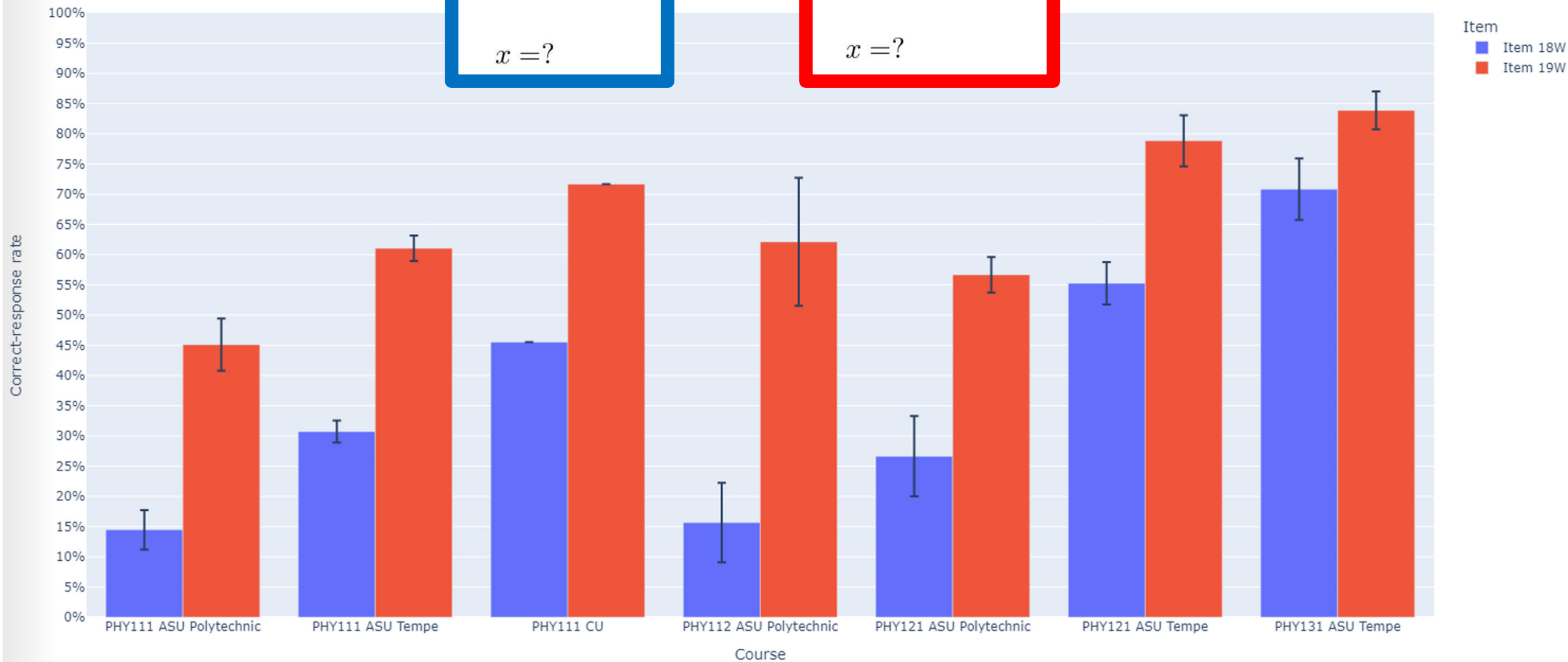
$$\begin{array}{l} 0.5y = 2x \\ 78.4 - y = 8x \end{array} \quad [\text{Solve for } x] \quad \text{Numeric Version} \quad 79\% \text{ correct } (N = 1205)$$

$$\begin{array}{l} cy = dx \\ a - y = bx \end{array} \quad [\text{Solve for } x] \quad \text{Symbolic Version} \quad 55\% \text{ correct } (N = 1202)$$

Course Averages

$$cy = dx$$
$$a - y = bx$$
$$x = ?$$

$$0.5y = 2x$$
$$78.4 - y = 8x$$
$$x = ?$$



## Conclusion:

Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by  $\approx 25\%$ .

Confusion can result from the *nature* of the symbols themselves

Solve for  $\theta$ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for  $x$ .

$$ax + b = cx + d$$

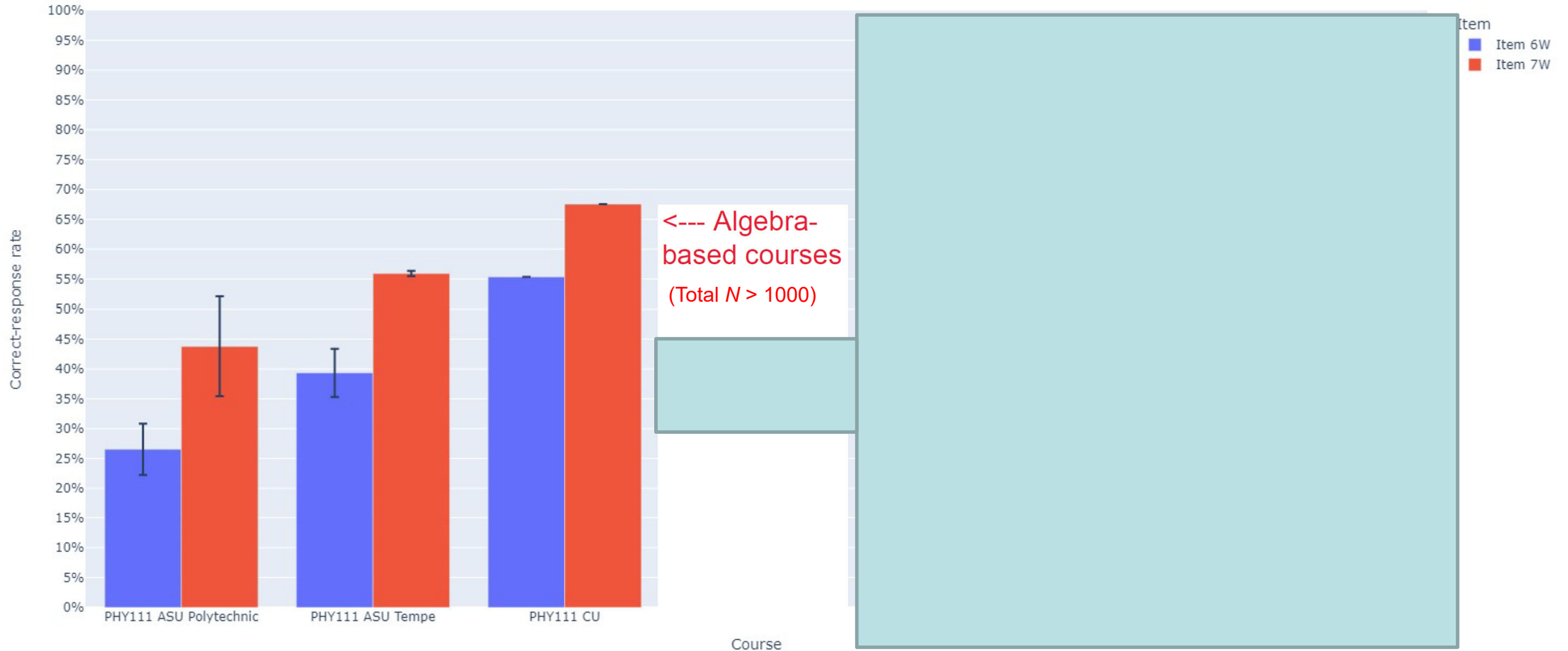
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Significantly lower correct-response rates on Greek-letter version in algebra-based courses

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$$ax + b = cx + d$$



# Implications of Findings

- Minor changes to problem properties can lead to vastly different correct-response rates.
- Certain specific elements of diagrams or terminology can divert students into long chains of unproductive reasoning.
- Use of symbols instead of numbers, or use of unfamiliar symbols, can significantly degrade student performance
- **To ensure consistency and reliability of assessment problems and instruments, research on students' responses to variations in problem properties is essential.**