What makes a physics problem hard? Research on problem difficulty

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What makes a problem hard(er)?

- Use of potentially misleading/distracting diagrammatic elements
- Inclusion of (potentially misleading) irrelevant or redundant information
- Use of symbols in place of numbers (e.g., m or μ for *mass*)
- Multiple relevant factors or variables in same problem (e.g., E, Q, W)
- Dependence on or use of unfamiliar or subtle assumptions or terms
  - e.g., *adiabatic*, *thermal reservoir*, *quasi-static*, *reversible, spontaneous*

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Investigating the impact of problem properties on introductory and advanced student responses to introductory thermodynamics conceptual problems

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Targeted Concepts (among others)

(*E*= internal energy; *W* = work done *by* system; *Q* = heat transfer *to* system)

- *E* is proportional to *T* for an ideal gas
- The sign of *ΔE* for an ideal gas is determined by whether the product *PV* is increasing or decreasing
- *W* is positive for an expansion

Information provided to students on first page:

Survey of Thermodynamic Processes and First and Second Laws (STPFaSL-Long)

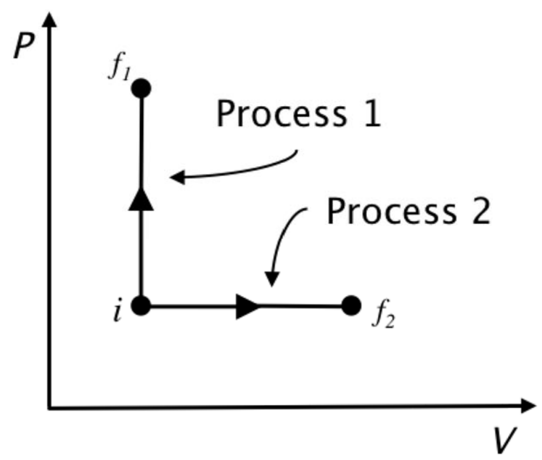
- Please select only one of the four choices, (a)-(d) or True/False for each of the questions.
- All temperatures T are absolute temperatures.
- All experiments involving a gas as the system are performed with a fixed amount of gas.
- The following equations may be useful for an ideal monatomic gas system where the symbols have the usual meaning: the internal energy $E_{int} = (3/2)NkT$ and $PV = NkT$.
- Thermal reservoirs are significantly larger than the system so that heat transfer between the system and the reservoir does not change the temperature of the reservoir.
- An adiabatic process is one in which there is no heat transfer between a system and its surroundings.
- The process described in questions 60-62 is quasi-static. A quasi-static process passes through a sequence of equilibrium states.

The following abbreviations are used throughout the survey:

- W = work done by the system.
- Q = net heat transfer to the system.

Also, Q_1, Q_2 in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes start in the same state i as shown on the PV diagram below. Process 1 is a constant volume (isochoric) process and process 2 is a constant pressure (isobaric) process. Answer the following six questions about the two processes:



(44) Which one of the following statements is correct about the change in internal energy of the gas in process 1?

a. There is no change in the internal energy of the gas in process 1.
b. The internal energy of the gas increases in process 1.
c. The internal energy of the gas decreases in process 1.
d. Not enough information.

(45) Which one of the following statements is correct about the change in internal energy of the gas in process 2?

a. There is no change in the internal energy of the gas in process 2.
b. The internal energy of the gas increases in process 2.
c. The internal energy of the gas decreases in process 2.
d. Not enough information.

[$N \approx 500$]

Answer to *both* #44 and #45: (b); *PV* increases so internal energy increases [$E_{int} = (3/2)\,NkT = (3/2)\,PV$].

Correct-response rate on Process 1, Calculus-based: 69%; Algebra-based: 70%

Correct-response rate on Process 2, Calculus-based: 43%; Algebra-based: 37% ⟵ *More incorrect answers on Process 2*

Interviews: 7 of 9 incorrect responses on Process 2 were justified by saying work done is positive so energy would decrease, thus ignoring the role of heat transfer.

For Process 2, the salience of the right-pointing horizontal arrow indicating that positive work is done lured students into flawed arguments regarding work and ignoring heat transfer.

The use of a horizontal arrow on a *PV* diagram (instead of an arrow pointed straight up) lured students into unproductive considerations regarding *work*, lowering the correct-response rate by 26-33%.

Targeted Concepts (among others)

(E= internal energy; W = work done *by* system; Q = heat transfer *to* system)

- E is proportional to T for an ideal gas
- The sign of ΔE for an ideal gas is determined by whether the product PV is increasing or decreasing
- W is positive for an expansion

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The following abbreviations are used throughout the survey:

- W = work done by the system.
- Q = net heat transfer to the system.

Also, Q_1, Q_2 in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

An ideal gas is allowed to undergo an isothermal expansion. Answer the following three questions about this process.

(62) Which one of the following statements is true about the work done by the gas in this process?

a. The work done by the gas is positive.
b. The work done by the gas is negative.
c. The work done by the gas is zero.
d. Not enough information.

Correct-response rate
Calculus-based: 65%; Algebra-based: 55%

Work = 0 responses
Calculus-based: 11%; Algebra-based: 16%

You perform an experiment with a gas such that it undergoes a reversible adiabatic expansion. Answer questions (1) - (3) below about this experiment.

(3) Which one of the following statements must be true for the work done by the gas that undergoes a reversible adiabatic expansion process?

a. The work done by the gas must be positive.
b. The work done by the gas must be negative.
c. The work done by the gas must be zero.
d. Not enough information.

Correct-response rate
Calculus-based: 54%; Algebra-based: 41%

Lower correct-response rate

Work = 0 responses
Calculus-based: 19%; Algebra-based: 27%

More $W = 0$ responses

[$N \approx 300$ for each class]

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More W = 0 responses

More Work = 0 responses on "reversible adiabatic" expansion question

Interviews: Most incorrect answers on #3 were justified by Q = 0 arguments

The replacement of the word *isothermal* by the term *reversible adiabatic* lowered the correct-response rate by 11-14%, even though both processes were described as an "expansion."

Symbolic Procedures

Confusion of symbolic meaning: Students perform worse on solving problems when symbols are used to represent common physical quantities in equations [Torigoe and Gladding, 2007; 2011)

Example [Multiple-choice questions; University of Illinois]:

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s? *[93% correct]*

Version #2: A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)? *[57% correct]*

Much worse!

➢ *Our results on "stripped-down" versions are analogous, although differences are smaller*

$$v^2 = v_0^2 + 2ad$$
$$v_0 = 0$$
$$a = \frac{v_1}{t_1}$$
$$v = \frac{v_1}{2}$$

$$d = ?$$

A. $d = v_1 t_1$ **B.** $d = \frac{v_1 t_1}{2}$ **C.** $d = \frac{v_1 t_1}{4}$ **D.** $\boxed{d = \frac{v_1 t_1}{8}}$ **E.** $d = \frac{v_1 t_1}{16}$

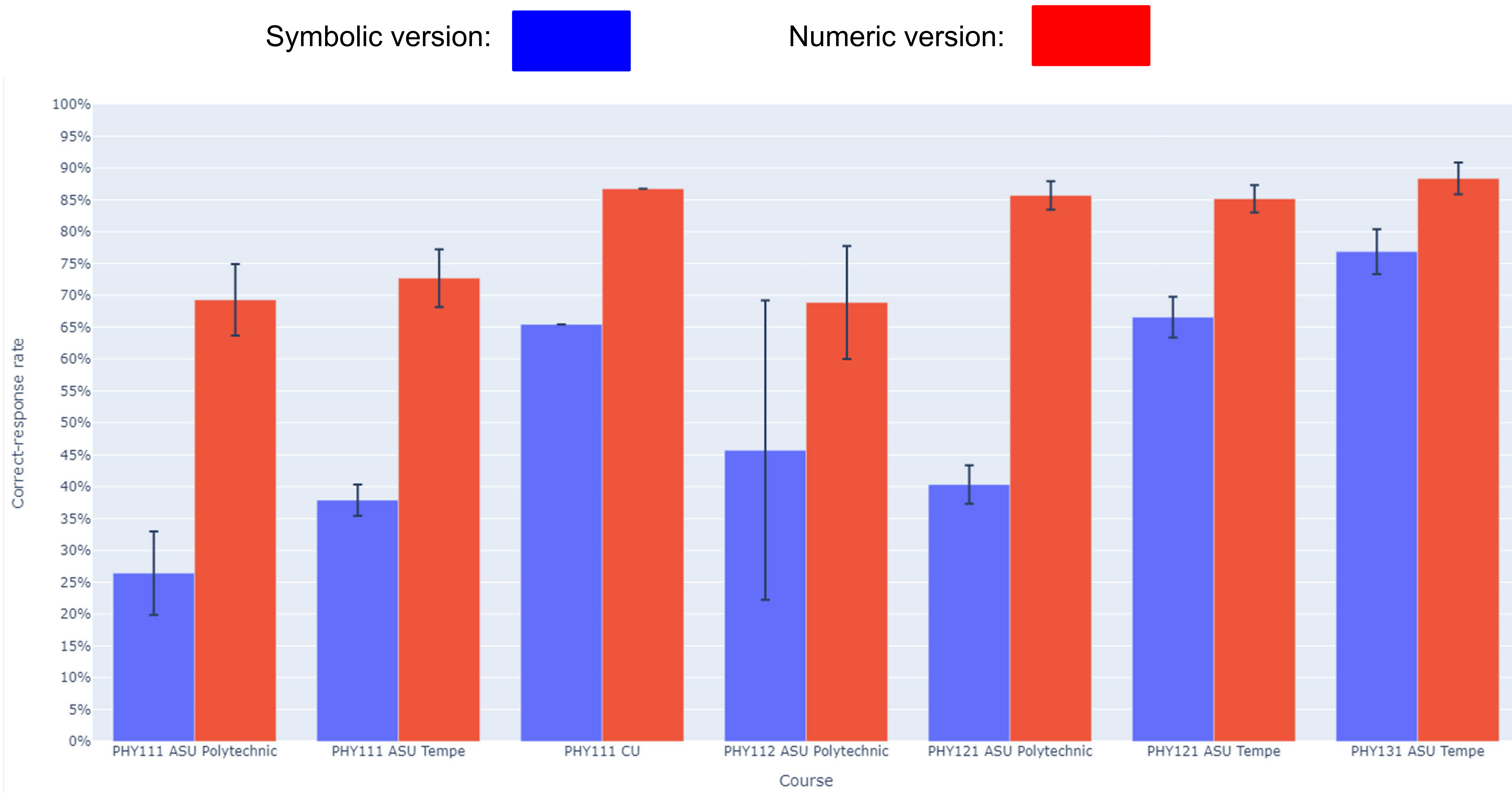
← Symbolic version

Numeric version →

$$v^2 = v_0^2 + 2ad$$
$$v_0 = 0$$
$$a = \frac{\Delta v}{\Delta t}$$
$$\Delta v = 60$$
$$\Delta t = 8$$
$$v = 30$$

$$d = ?$$

A. $d = 30$ **B.** $\boxed{d = 60}$ **C.** $d = 120$ **D.** $d = 240$ **E.** $d = 480$


Symbolic version:
Numeric version:
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Correct-response rate
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe
Course

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)

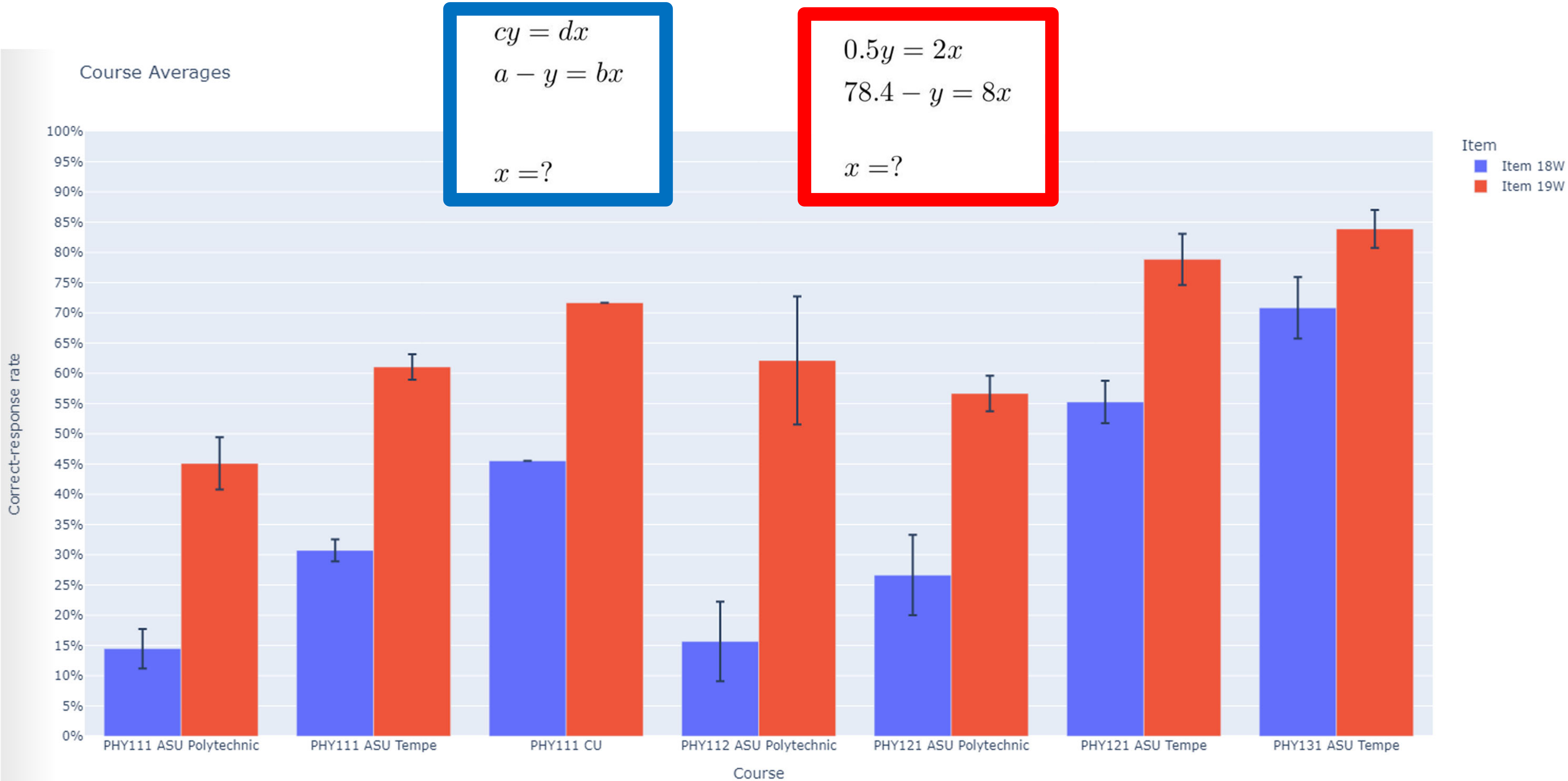
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	31% correct (N = 372)

Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)

Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	55% correct (N = 1202)


Course Averages
cy = dx
a − y = bx
x =?
0.5y = 2x
78.4 − y = 8x
x =?
Item
Item 18W
Item 19W
Correct-response rate
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe
Course

Conclusion:
Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by ≈25%.

Confusion can result from the *nature* of the symbols themselves

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for x.

$$ax + b = cx + d$$

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Significantly lower correct-response rates on Greek-letter version in algebra-based courses

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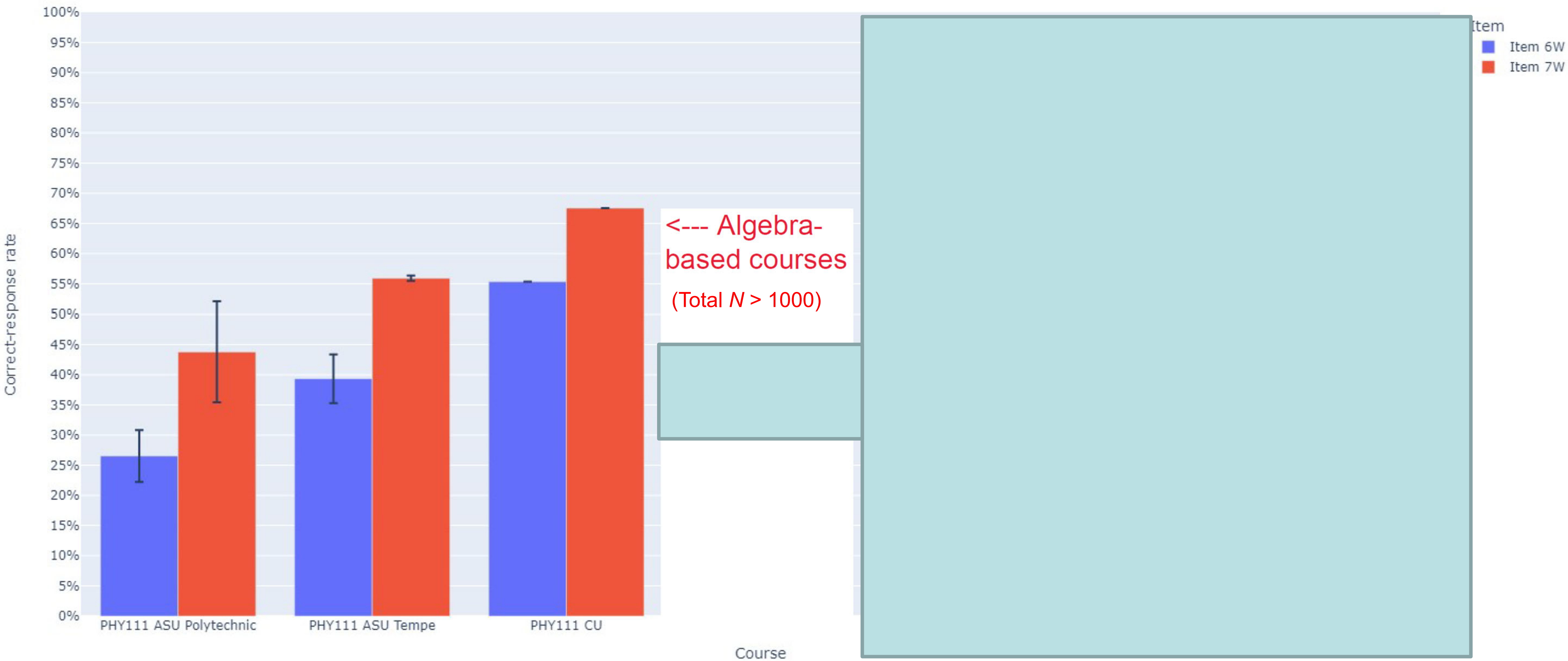

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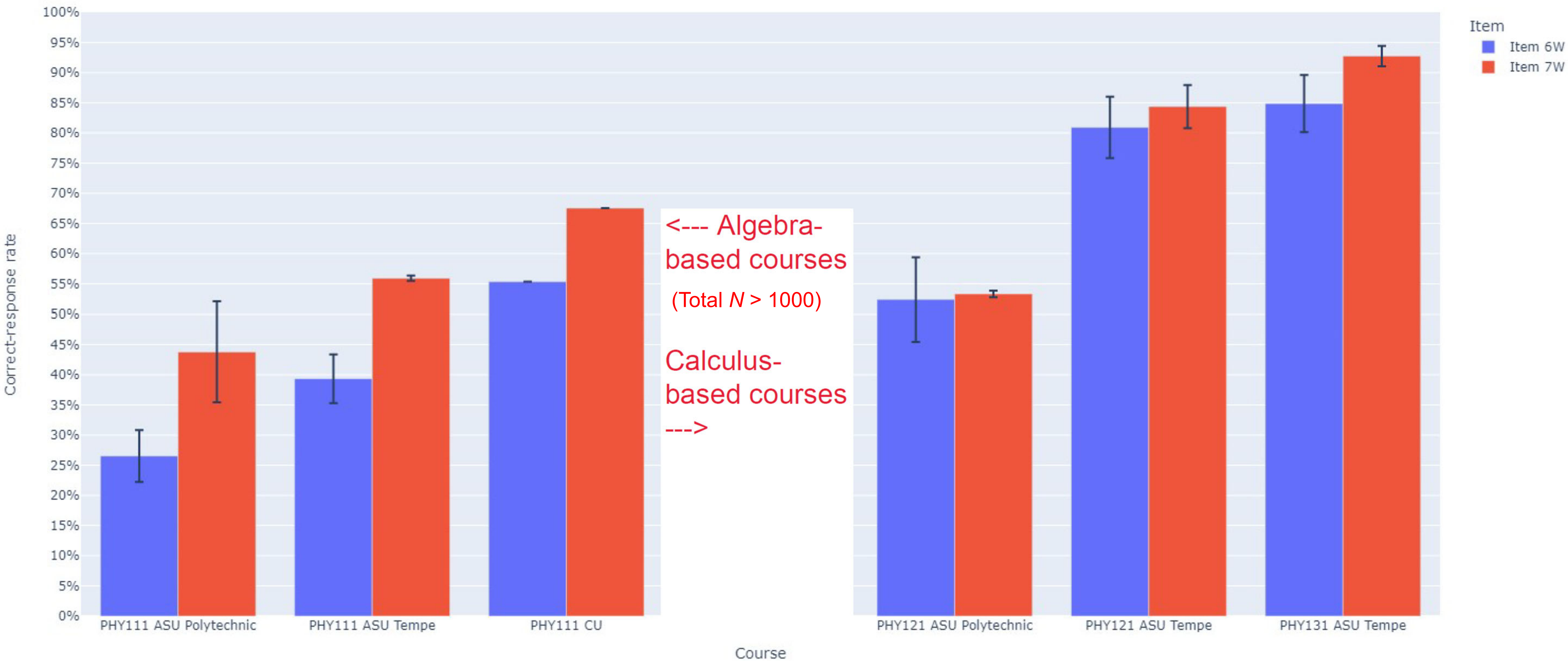

Implications of Findings

- Minor changes to problem properties can lead to vastly different correct-response rates.
- Certain specific elements of diagrams or terminology can divert students into long chains of unproductive reasoning.
- Use of symbols instead of numbers, or use of unfamiliar symbols, can significantly degrade student performance
- Focused instructional guidance may be needed to aid students in addressing these challenges.