How can we assess students' knowledge when small differences in problem properties can significantly alter students' responses?

David E. Meltzer
Arizona State University

Supported in part by NSF DUE #1504986 and #1914712

Overview: Experts’ vs. Learners’ Knowledge

- An “expert” can see through superficial differences between problem properties (e.g., diagrams, wording, physical context) and focus sharply on the underlying concept, usually proceeding smoothly towards a productive solution pathway.

- A “learner” has incomplete and imperfect knowledge, and can often be distracted or misled by irrelevant or insignificant problem properties, leading to incorrect responses.

Overview: Experts’ vs. Learners’ Knowledge

- We often implicitly assume that “small” changes in problem properties lead to small changes in students’ responses, therefore assuming that large differences in students’ responses to (apparently) identical problems signify large differences in student understanding.

- Suppose, by contrast, that small changes in problem properties can lead to ***large*** differences in students’ responses—“nonlinear response,” so to speak. Then, consistency of students’ responses between different assessment problems and instruments could be reduced. How, then, are we to assess students’ true understanding of the underlying concepts?

What “non-conceptual” properties can increase problem difficulty?

- Use of potentially misleading/distracting diagrammatic elements
- Inclusion of (potentially misleading) irrelevant or redundant information
- Use of symbols in place of numbers (e.g., *m* or *μ* for *mass*)
- Multiple relevant factors or variables in same problem (e.g., *E*, *Q*, *W*)
- Dependence on or use of unfamiliar or subtle assumptions or terms
  - e.g., *adiabatic*, *thermal reservoir*, *quasi-static*, *reversible, spontaneous*

PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **21,** 010127 (2025)

Investigating the impact of problem properties on introductory and advanced student responses to introductory thermodynamics conceptual problems

Mary Jane Brundage,[1] David E. Meltzer,[2] and Chandralekha Singh[1]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pittsburgh 15260, USA*
[2]*College of Integrative Sciences and Arts, Arizona State University, Mesa, Arizona 85212, USA*

(Received 28 August 2023; accepted 10 February 2025; published 28 March 2025)

Exploring students' thinking when solving thermodynamics problems

- **78-item Multiple-choice instrument:** "Survey of Thermodynamic Processes and First and Second Laws (STPFaSL-Long)"
  - administered after instruction in standard algebra- and calculus-based lecture courses
- Each individual thermodynamics concept was targeted by 2-5 different problems
- Problems differ from each other by:
  - using diverse physical settings and scenarios
  - very minor changes in wording or in diagrammatic features
  - including diverse potentially distracting features
- Sample size ranged from 320-550 (varied by problem)
- With these sample sizes, differences in correct-response rates ≈15% or greater are statistically significant, generally with $p<0.001$ and effect size > 0.3
- **Interview data**: Interviews carried out with 17 students (11 introductory; 6 upper-level)

Targeted Concepts (among others)

(*E*= internal energy; *W* = work done *by* system; *Q* = heat transfer *to* system)

- *E* is proportional to *T* for an ideal gas
- The sign of ΔE for an ideal gas is determined by whether the product *PV* is increasing or decreasing
- *W* is positive for an expansion
- In a reversible isothermal process, $Q \neq 0$ and the sign of *Q* is determined by whether volume is increasing or decreasing

Targeted Concepts (among others)

(*E*= internal energy; *W* = work done *by* system; *Q* = heat transfer *to* system)

- *E* is proportional to *T* for an ideal gas
- The sign of ΔE for an ideal gas is determined by whether the product *PV* is increasing or decreasing
- *W* is positive for an expansion
- In a reversible isothermal process, $Q \neq 0$ and the sign of *Q* is determined by whether volume is increasing or decreasing

Information provided to students on first page:

Survey of Thermodynamic Processes and First and Second Laws (STPFaSL-Long)

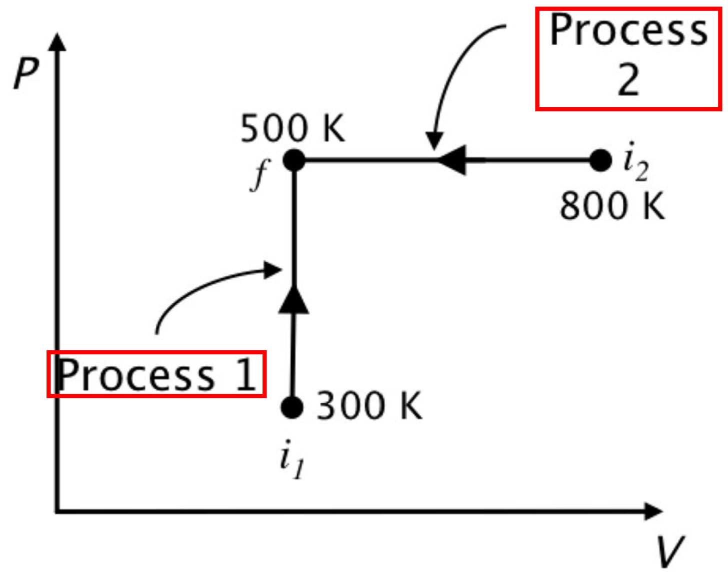
- Please select only one of the four choices, (a)-(d) or True/False for each of the questions.
- All temperatures T are absolute temperatures.
- All experiments involving a gas as the system are performed with a fixed amount of gas.
- The following equations may be useful for an ideal monatomic gas system where the symbols have the usual meaning: the internal energy $E_{int} = (3/2)NkT$ and $PV = NkT$.
- Thermal reservoirs are significantly larger than the system so that heat transfer between the system and the reservoir does not change the temperature of the reservoir.
- An adiabatic process is one in which there is no heat transfer between a system and its surroundings.
- The process described in questions 60-62 is quasi-static. A quasi-static process passes through a sequence of equilibrium states.

The following abbreviations are used throughout the survey:

- W = work done by the system.
- Q = net heat transfer to the system.

Also, Q_1, Q_2 in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes end in the same state shown on the PV diagram below. Process 1 is a constant volume process starting at 300K at point i_1 and ending at point f at 500K whereas process 2 is a constant pressure process starting at 800K at point i_2 and ending at point f at 500K. Answer the following two questions about these experiments:



(33) Which one of the following statements is true about the change in the internal energy of the gas for process 1?

a. Internal energy does not change for process 1.
b. Internal energy increases for process 1.
c. Internal energy decreases for process 1.
d. Not enough information.

Answer: (b); temperature increases so internal energy increases.
Correct-response rate; Calculus-based: 76%; Algebra-based: 81%

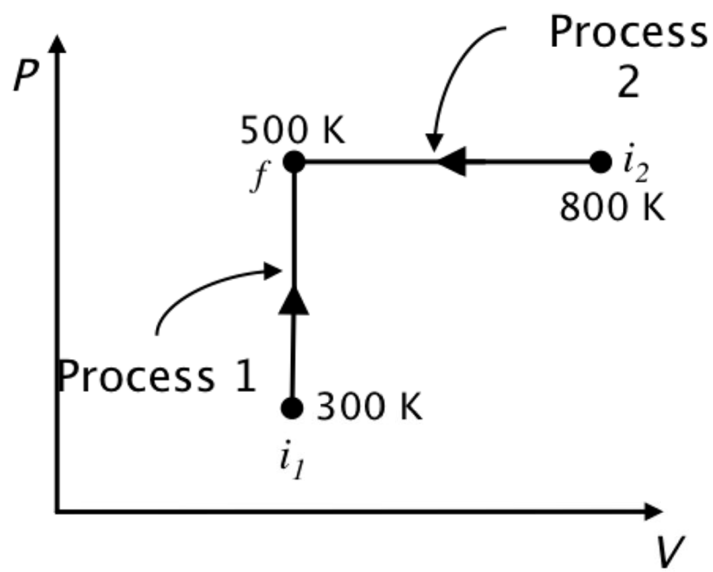
(34) Which one of the following statements is true about the change in the internal energy of the gas for process 2?

a. Internal energy does not change for process 2.
b. Internal energy increases for process 2.
c. Internal energy decreases for process 2.
d. Not enough information.

Many more errors on #34 (Process 2) →

Answer: (c); temperature decreases so internal energy decreases.
Correct-response rate; Calculus-based: 58%; Algebra-based: 55%

Why did students make significantly more errors on #34?



Interviews: 3 of 6 students who gave incorrect answer on #34 (Process 2) argued "Work done is negative so internal energy will increase"

This would be true for an adiabatic *process; however, students have completely ignored the role of heat transfer (Q < 0 in this case).*

Students were distracted by work considerations; this was also true for Process 1 (zero work) but not, apparently, to the same degree as in Process 2.

The salience of the negative-work feature in Process 2 seems to have played a key role in students' thinking.

(34) Which one of the following statements is true about the change in the internal energy of the gas for process 2?

a. Internal energy does not change for process 2.
b. Internal energy increases for process 2.
c. Internal energy decreases for process 2.
d. Not enough information.

Answer: (c); temperature decreases so internal energy decreases.

Correct-response rate; Calculus-based: 58%; Algebra-based: 55%

The use of a horizontal arrow on a *PV* diagram (instead of an arrow pointed straight up) lured some students into unproductive considerations regarding *work*, lowering the correct-response rate by 18-26%.

Targeted Concepts (among others)

(*E*= internal energy; *W* = work done *by* system; *Q* = heat transfer *to* system)

- *E* is proportional to *T* for an ideal gas
- The sign of *ΔE* for an ideal gas is determined by whether the product *PV* is increasing or decreasing
- *W* is positive for an expansion
- In a reversible isothermal process, *Q ≠ 0* and the sign of *Q* is determined by whether volume is increasing or decreasing

Information provided to students on first page:

Survey of Thermodynamic Processes and First and Second Laws (STPFaSL-Long)

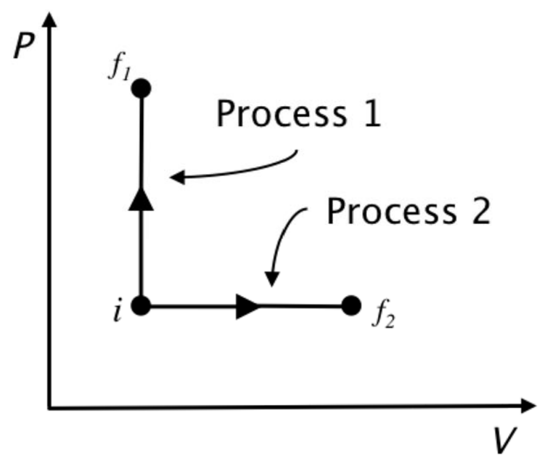
- Please select only one of the four choices, (a)-(d) or True/False for each of the questions.
- All temperatures T are absolute temperatures.
- All experiments involving a gas as the system are performed with a fixed amount of gas.
- The following equations may be useful for an ideal monatomic gas system where the symbols have the usual meaning: the internal energy $E_{int} = (3/2)NkT$ and $PV = NkT$.
- Thermal reservoirs are significantly larger than the system so that heat transfer between the system and the reservoir does not change the temperature of the reservoir.
- An adiabatic process is one in which there is no heat transfer between a system and its surroundings.
- The process described in questions 60-62 is quasi-static. A quasi-static process passes through a sequence of equilibrium states.

The following abbreviations are used throughout the survey:

- W = work done by the system.
- Q = net heat transfer to the system.

Also, Q_1, Q_2 in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

You carry out two experiments each with one mole of an ideal monatomic gas such that both processes start in the same state i as shown on the PV diagram below. Process 1 is a constant volume (isochoric) process and process 2 is a constant pressure (isobaric) process. Answer the following six questions about the two processes:



(44) Which one of the following statements is correct about the change in internal energy of the gas in process 1?

a. There is no change in the internal energy of the gas in process 1.
b. The internal energy of the gas increases in process 1.
c. The internal energy of the gas decreases in process 1.
d. Not enough information.

(45) Which one of the following statements is correct about the change in internal energy of the gas in process 2?

a. There is no change in the internal energy of the gas in process 2.
b. The internal energy of the gas increases in process 2.
c. The internal energy of the gas decreases in process 2.
d. Not enough information.

[N ≈500]

Answer to *both* #44 and #45: (b); *PV* increases so internal energy increases [$E_{int} = (3/2)\,NkT = (3/2)\,PV$].

Correct-response rate on Process 1, Calculus-based: 69%; Algebra-based: 70%

Correct-response rate on Process 2, Calculus-based: 43%; Algebra-based: 37% ← *More incorrect answers on Process 2*

Interviews: 7 of 9 incorrect responses on Process 2 were justified by saying work done is positive so energy would decrease, thus ignoring the role of heat transfer.

For Process 2, the salience of the right-pointing horizontal arrow indicating that positive work is done lured students into flawed arguments regarding work and ignoring heat transfer.

The use of a horizontal arrow on a *PV* diagram (instead of an arrow pointed straight up) lured students into unproductive considerations regarding *work*, lowering the correct-response rate by 26-33%.

Targeted Concepts (among others)

(*E*= internal energy; *W* = work done *by* system; *Q* = heat transfer *to* system)

- *E* is proportional to *T* for an ideal gas
- The sign of *ΔE* for an ideal gas is determined by whether the product *PV* is increasing or decreasing
- *W* is positive for an expansion
- In a reversible isothermal process, $Q \neq 0$ and the sign of *Q* is determined by whether volume is increasing or decreasing

Information provided to students on first page:

Survey of Thermodynamic Processes and First and Second Laws (STPFaSL-Long)

- Please select only one of the four choices, (a)-(d) or True/False for each of the questions.
- All temperatures T are absolute temperatures.
- All experiments involving a gas as the system are performed with a fixed amount of gas.
- The following equations may be useful for an ideal monatomic gas system where the symbols have the usual meaning: the internal energy $E_{int} = (3/2)NkT$ and $PV = NkT$.
- Thermal reservoirs are significantly larger than the system so that heat transfer between the system and the reservoir does not change the temperature of the reservoir.
- An adiabatic process is one in which there is no heat transfer between a system and its surroundings.
- The process described in questions 60-62 is quasi-static. A quasi-static process passes through a sequence of equilibrium states.

The following abbreviations are used throughout the survey:

- W = work done by the system.
- Q = net heat transfer to the system.

Also, Q_1, Q_2 in a particular problem will refer to the net heat transfer to the system in process 1 and process 2, respectively, etc.

An ideal gas is allowed to undergo an isothermal expansion. Answer the following three questions about this process.

(62) Which one of the following statements is true about the work done by the gas in this process?

a. The work done by the gas is positive.
b. The work done by the gas is negative.
c. The work done by the gas is zero.
d. Not enough information.

Correct-response rate
Calculus-based: 65%; Algebra-based: 55%

Work = 0 responses
Calculus-based: 11%; Algebra-based: 16%

You perform an experiment with a gas such that it undergoes a reversible adiabatic expansion. Answer questions (1) - (3) below about this experiment.

(3) Which one of the following statements must be true for the work done by the gas that undergoes a reversible adiabatic expansion process?

a. The work done by the gas must be positive.
b. The work done by the gas must be negative.
c. The work done by the gas must be zero.
d. Not enough information.

Correct-response rate
Calculus-based: 54%; Algebra-based: 41%

Lower correct-response rate

Work = 0 responses
Calculus-based: 19%; Algebra-based: 27%

More W = 0 responses

[*N* ≈300 for each class]

An ideal gas is allowed to undergo an isothermal expansion. Answer the following three questions about this process.

(62) Which one of the following statements is true about the work done by the gas in this process?

a. The work done by the gas is positive.
b. The work done by the gas is negative.
c. The work done by the gas is zero.
d. Not enough information.

Correct-response rate

Calculus-based: 65%; Algebra-based: 55%

Work = 0 responses

Calculus-based: 11%; Algebra-based: 16%

You perform an experiment with a gas such that it undergoes a reversible adiabatic expansion. Answer questions (1) - (3) below about this experiment.

(3) Which one of the following statements must be true for the work done by the gas that undergoes a reversible adiabatic expansion process?

a. The work done by the gas must be positive.
b. The work done by the gas must be negative.
c. The work done by the gas must be zero.
d. Not enough information.

Correct-response rate

Calculus-based: 54%; Algebra-based: 41%

Lower correct-response rate

Work = 0 responses

Calculus-based: 19%; Algebra-based: 27%

More W = 0 responses

More Work = 0 responses on "reversible adiabatic" expansion question

Interviews: Most incorrect answers on #3 were justified by Q = 0 arguments

The replacement of the word *isothermal* by the term *reversible adiabatic* lowered the correct-response rate by 11-14%, even though both processes were described as an “expansion.”

Targeted Concepts (among others)

(E= internal energy; W = work done *by* system; Q = heat transfer *to* system)

- E is proportional to T for an ideal gas
- The sign of ΔE for an ideal gas undergoing an isochoric or isobaric process is determined by whether pressure or volume are increasing
- W is positive for an expansion whether adiabatic or isothermal
- In a reversible isothermal process, $Q \neq 0$ and the sign of Q is determined by whether volume is increasing or decreasing

An ideal gas is allowed to undergo an isothermal expansion. Answer the following three questions about this process.

Q–W = ΔE = 0 since ΔT=0, so Q = W > 0

(60) Which one of the following statements is true about the heat transfer in this process?

a. There is no net heat transfer between the gas and its environment.

b. There is net heat transfer to the gas.

c. There is net heat transfer away from the gas.

d. Not enough information.

Correct-response rate

Calculus-based: 43%; Algebra-based: 44%

Q = 0 responses

Calculus-based: 30%; Algebra-based: 37%

You perform an experiment with an ideal monatomic gas such that it undergoes a reversible isothermal compression. Answer questions (4) and (5) below about this experiment.

Q–W = ΔE = 0 since ΔT=0, so Q = W < 0

(4) Which one of the following is true about the net heat transfer during the reversible isothermal compression?

a. There is net heat transfer to the gas.

b. There is net heat transfer away from the gas.

c. There is no net heat transfer to or from the gas.

d. Not enough information.

Correct-response rate

Calculus-based: 30%; Algebra-based: 21%

Q = 0 responses

Calculus-based: 50%; Algebra-based: 59%

Many more Q = 0 responses on “reversible compression” question, justified by “process is isothermal” arguments

General Findings

- Horizontal process arrows on *PV* diagrams often triggered unproductive lines of reasoning regarding work
- Certain terms may trigger unproductive lines of reasoning (e.g., *adiabatic*)
- Students tend to focus attention on the “most salient” variable (such as heat in an adiabatic process) while ignoring other variables

Symbolic Procedures

Confusion of symbolic meaning: Students perform worse on solving problems when symbols are used to represent common physical quantities in equations [Torigoe and Gladding, 2007; 2011)

Example [Multiple-choice questions; University of Illinois]:

Version #1: A car can go from 0 to 60 m/s in 8 s. At what distance *d* from the start at rest is the car traveling 30 m/s? *[93% correct]*

Version #2: A car can go from 0 to v_1 in t_1 seconds. At what distance *d* from the start at rest is the car traveling (v_1/2)? *[57% correct]*

Much worse!

➢ *Our results on "stripped-down" versions are analogous, although differences are smaller*

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{v_1}{t_1}$

$v = \frac{v_1}{2}$

Symbolic version

$d = ?$

A. $d = v_1 t_1$ **B.** $d = \frac{v_1 t_1}{2}$ **C.** $d = \frac{v_1 t_1}{4}$ **D.** $\boxed{d = \frac{v_1 t_1}{8}}$ **E.** $d = \frac{v_1 t_1}{16}$

Numeric version

$v^2 = v_0^2 + 2ad$

$v_0 = 0$

$a = \frac{\Delta v}{\Delta t}$

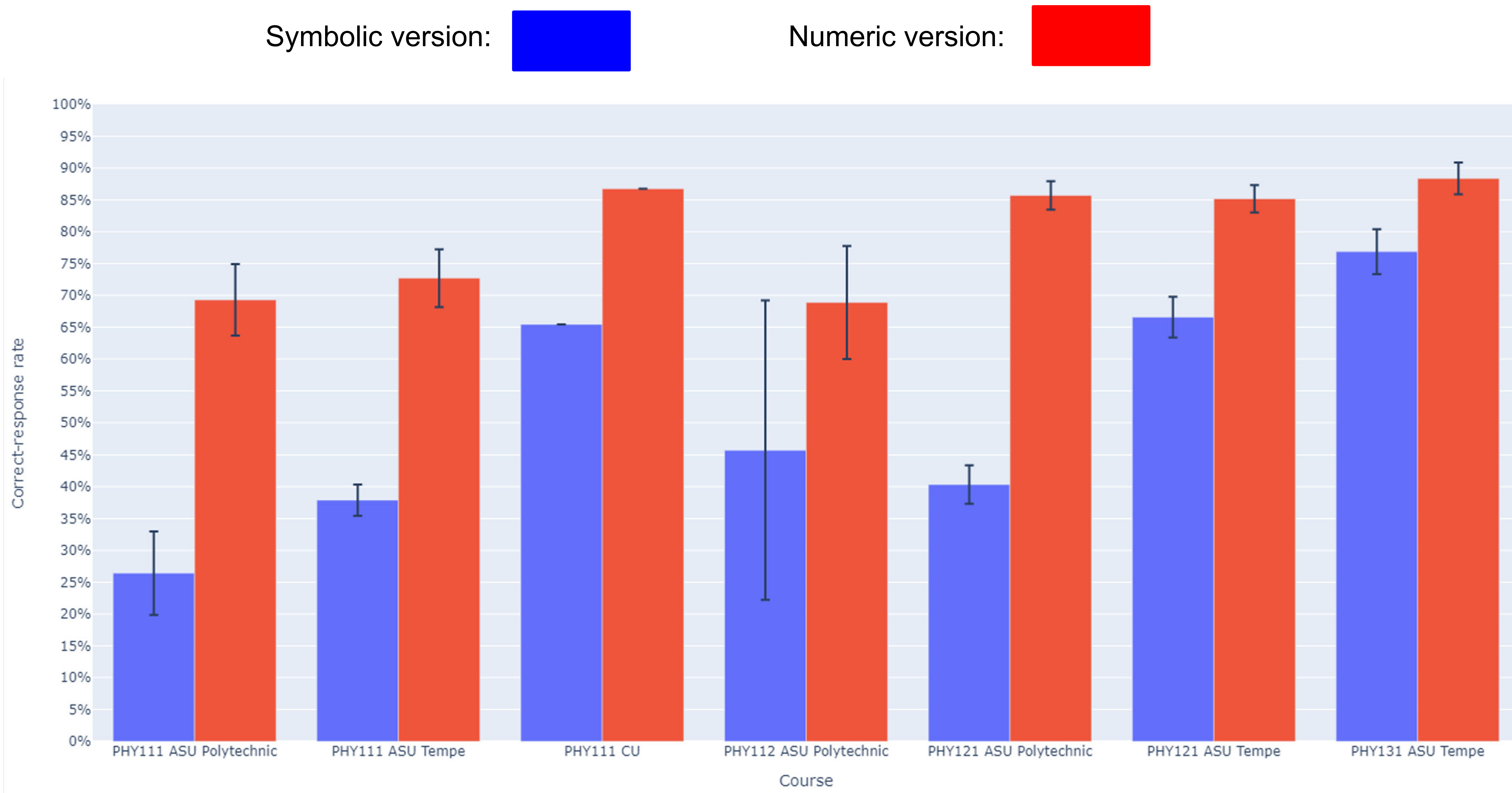
$\Delta v = 60$

$\Delta t = 8$

$v = 30$

$d = ?$

A. $d = 30$ **B.** $\boxed{d = 60}$ **C.** $d = 120$ **D.** $d = 240$ **E.** $d = 480$


Symbolic version:
Numeric version:
Correct-response rate
Course
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)

Algebra: Simultaneous Equations (algebra-based course, ASU-T)

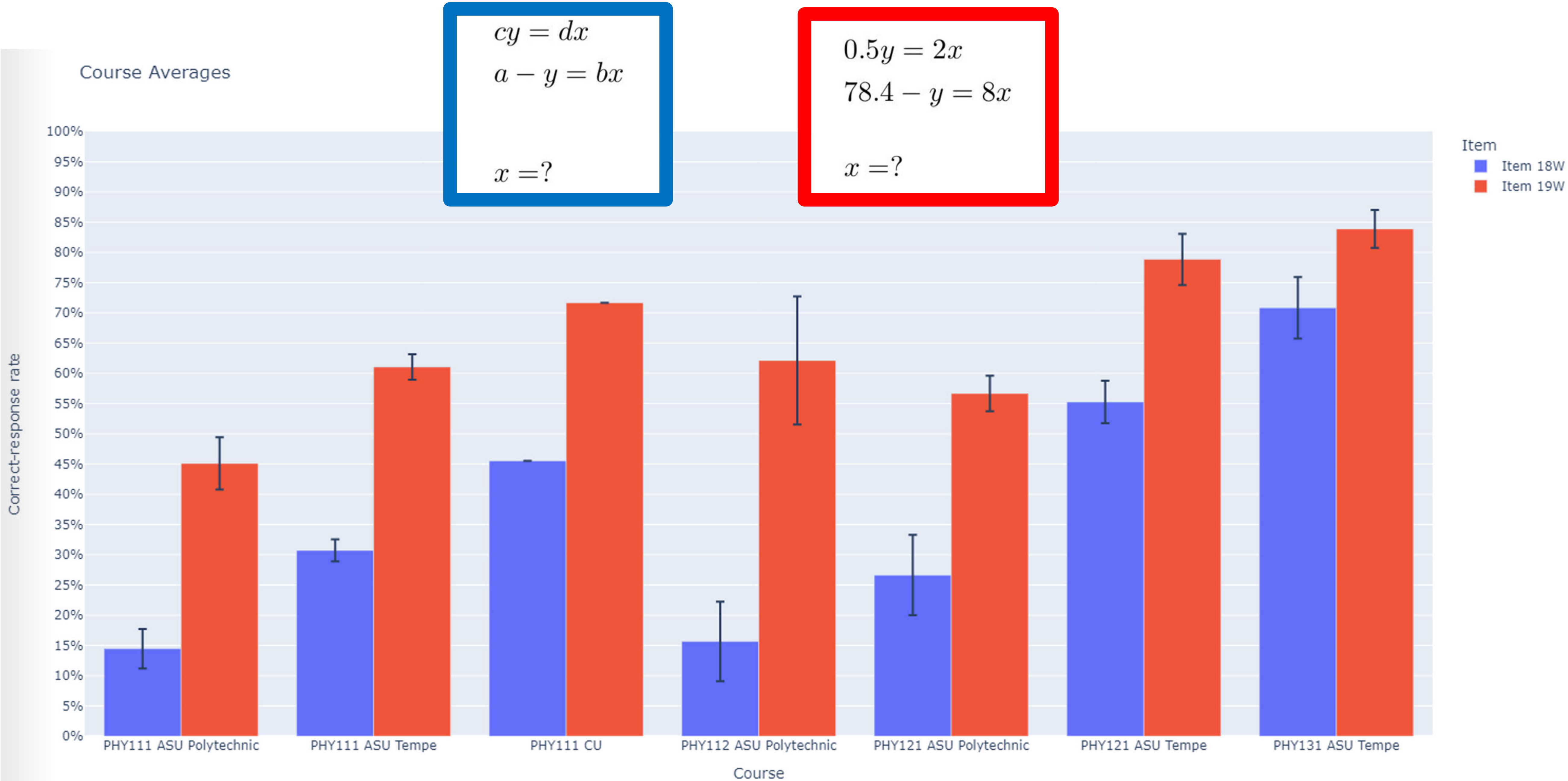
$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	61% correct (N = 470)
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	31% correct (N = 372)

Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)

Algebra: Simultaneous Equations (calculus-based course, ASU-T)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1205)
$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	55% correct (N = 1202)



$cy = dx$
$a - y = bx$
$x = ?$
$0.5y = 2x$
$78.4 - y = 8x$
$x = ?$
Course Averages
Correct-response rate
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Item
Item 18W
Item 19W
PHY111 ASU Polytechnic
PHY111 ASU Tempe
PHY111 CU
PHY112 ASU Polytechnic
PHY121 ASU Polytechnic
PHY121 ASU Tempe
PHY131 ASU Tempe
Course

Conclusion:
Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by ≈25%.

Confusion can result from the *nature* of the symbols themselves

Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for x**.**

$$ax + b = cx + d$$

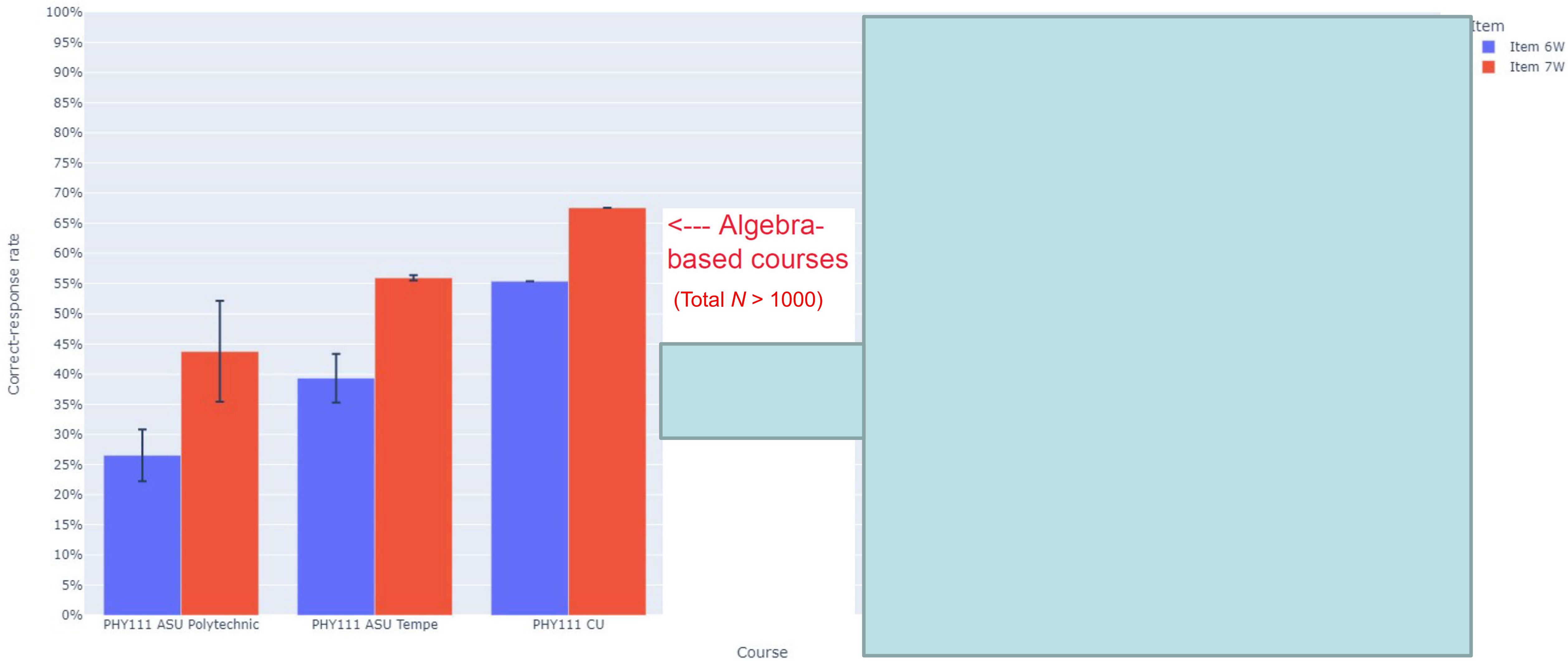
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x.

$$ax + b = cx + d$$



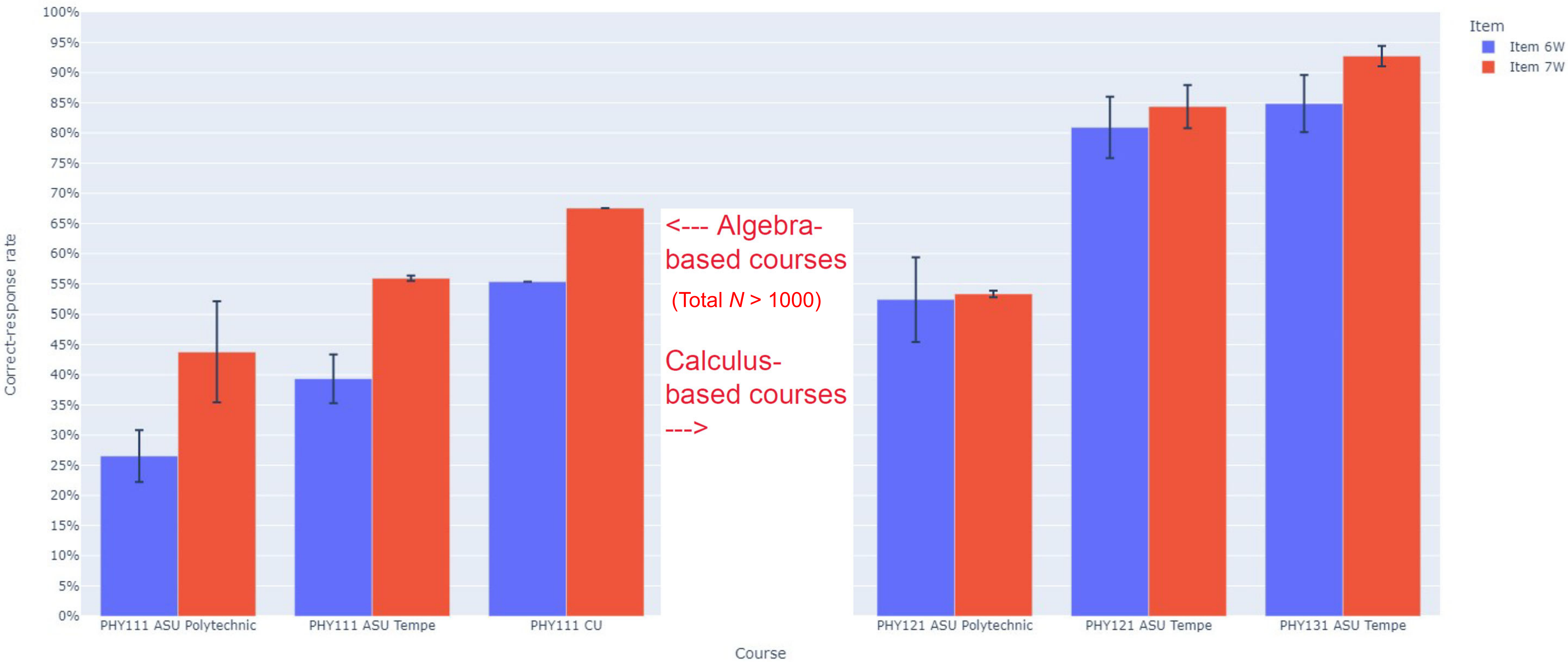
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x.

$$ax + b = cx + d$$



Implications of Findings

- Minor changes to problem properties can lead to vastly different correct-response rates.
- Certain specific elements of diagrams or terminology can divert students into long chains of unproductive reasoning.
- Use of symbols instead of numbers, or use of unfamiliar symbols, can significantly degrade student performance
- Focused instructional guidance may be needed to aid students in addressing these challenges.
- **To ensure consistency and reliability of assessment problems and instruments, research on students' responses to variations in problem properties is essential.**