

Investigation into the mathematical preparation of introductory physics students

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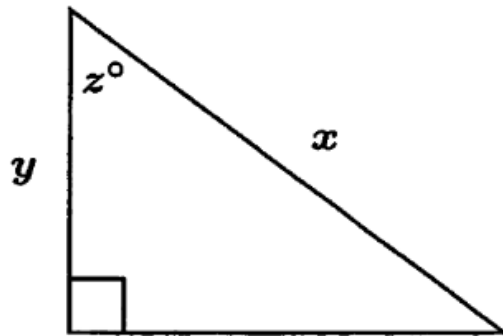
Data Sources

- Diagnostic pretests covering pre-college mathematics given to over 7000 introductory physics students (non-credit; calculators allowed):
- More than 80 one-on-one problem-solving interviews
- Pre-instruction tests of scientific reasoning skill and physics concept knowledge.

Examples of Test Items

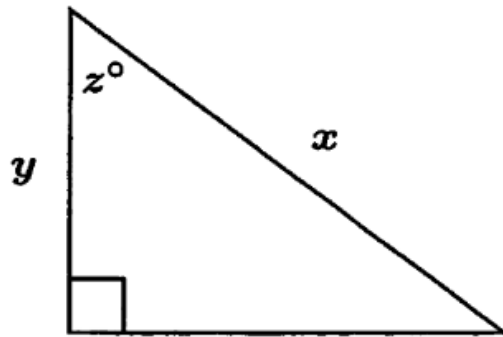
Find Unknown Side

1. What is the length of side x ?



Find Unknown Side

1. What is the length of side x ?

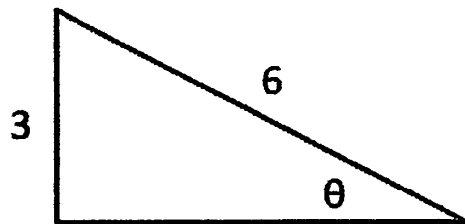


$$\cos z^\circ = \frac{y}{x}$$

$$x = \frac{y}{\cos z^\circ}$$

Find Unknown Angle

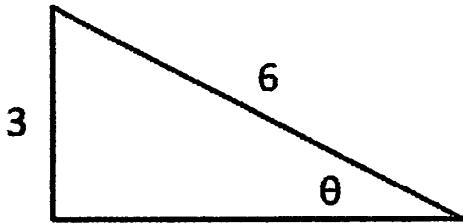
3.



What is the value of θ ?

Find Unknown Angle

3.



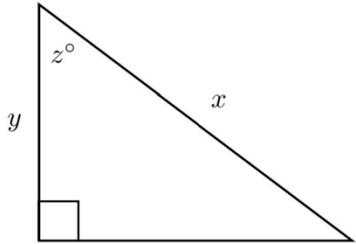
What is the value of θ ?

$$\sin^{-1}(\theta) = \sin^{-1}\left(\frac{3}{6}\right)$$
$$\theta = 30^\circ$$

Correct-response rates (written, free-response)

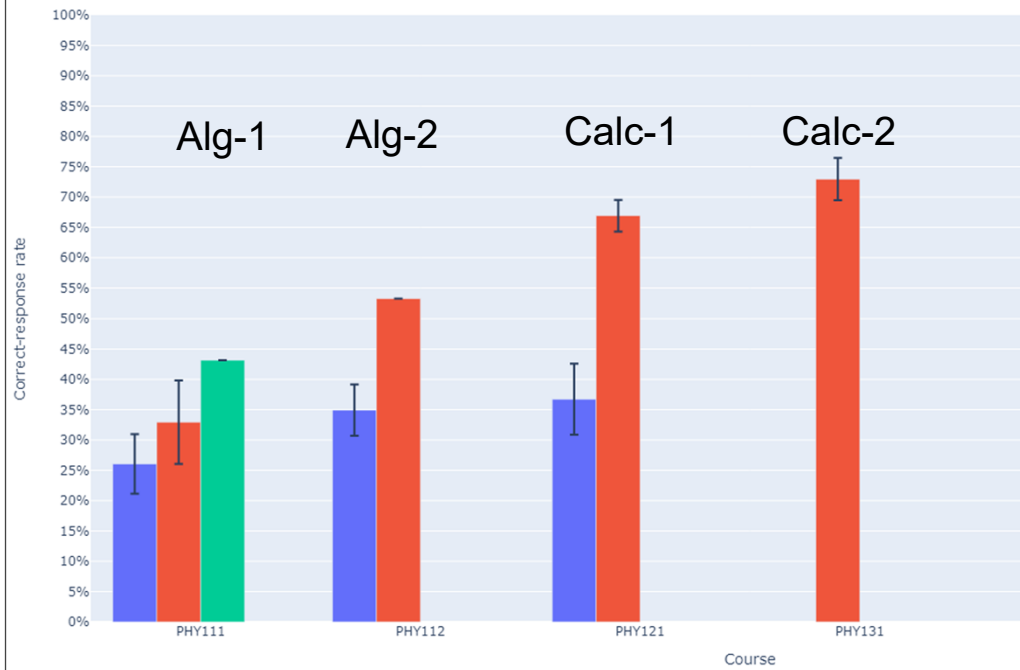
(36 classes; $N > 3000$)

What is the length of side x ?



Campus

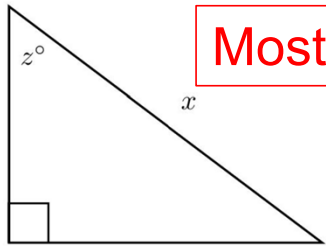
- ASU Polytechnic
- ASU Tempe
- CU



Correct-response rates (written, free-response)

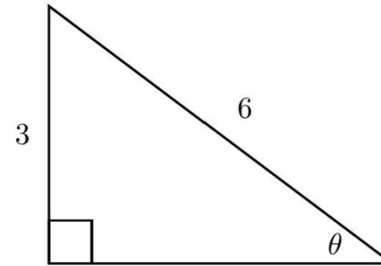
(36 classes; $N > 3000$)

What is the length of side x ?



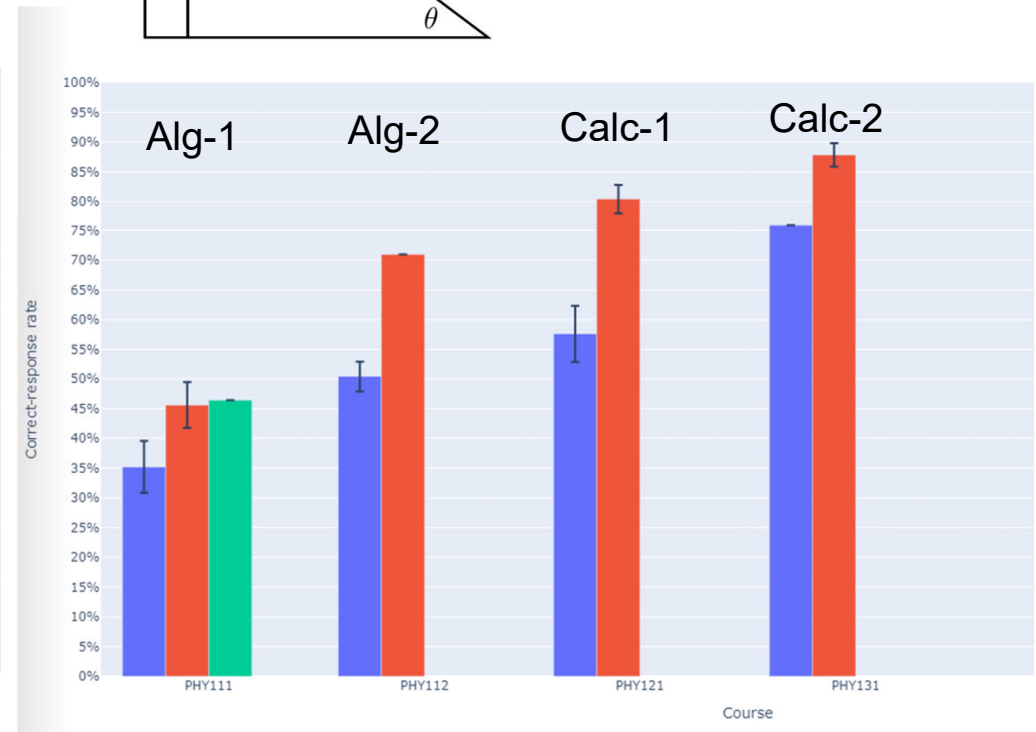
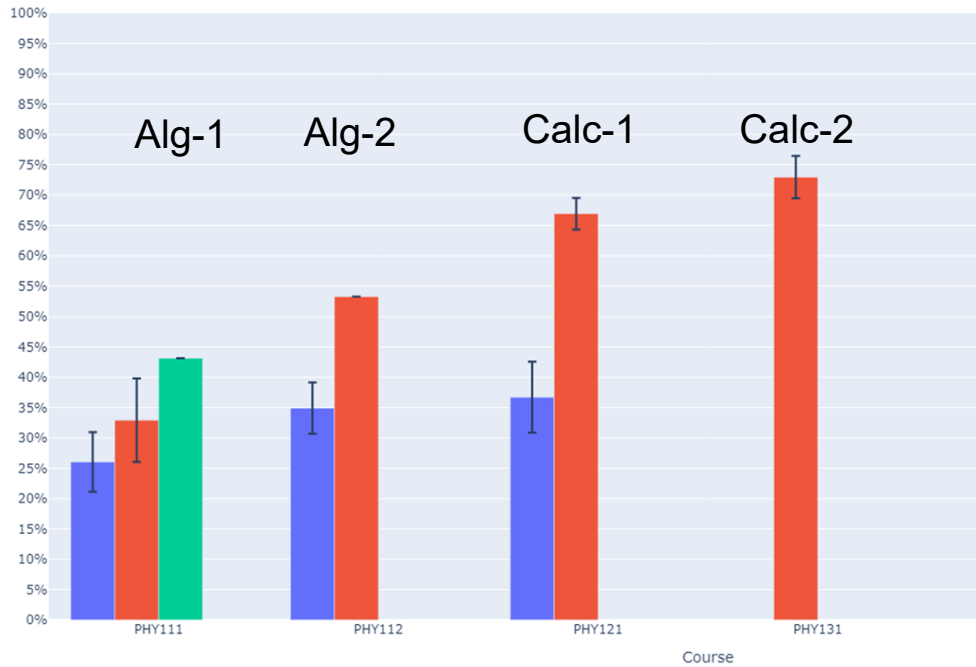
Most results in the 30-70% range

What is the value of θ ?

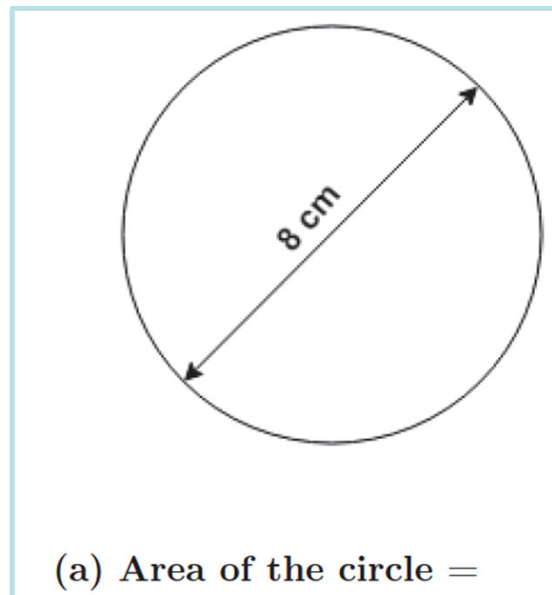


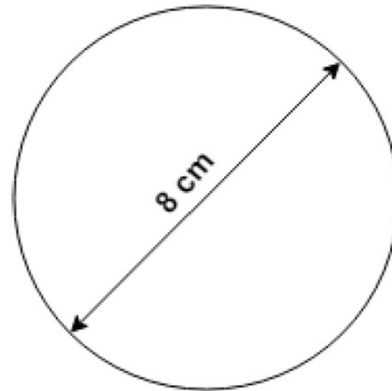
Campus

- ASU Polytechnic
- ASU Tempe
- CU



Find Area





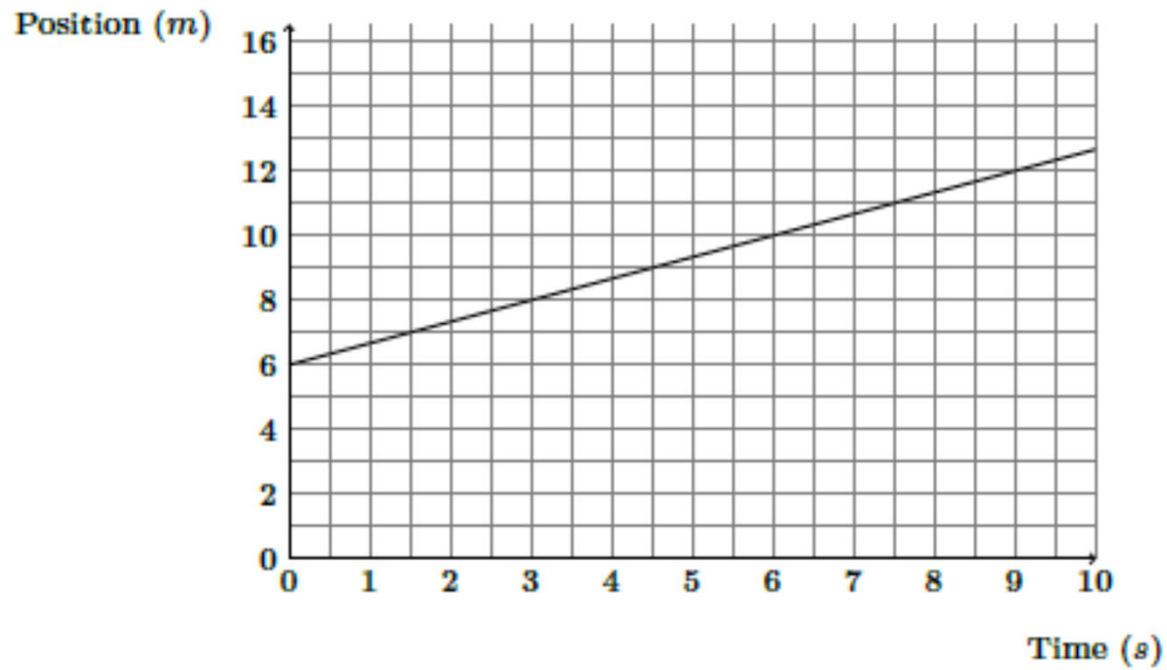
(a) Area of the circle =

Area of Circle: Algebra- and Calculus-based courses combined, 2018

	<i>N</i>	Numerically correct	Correct with correct units
ASU-Polytechnic	250	57%	29%
ASU-Tempe	1086	76%	45%

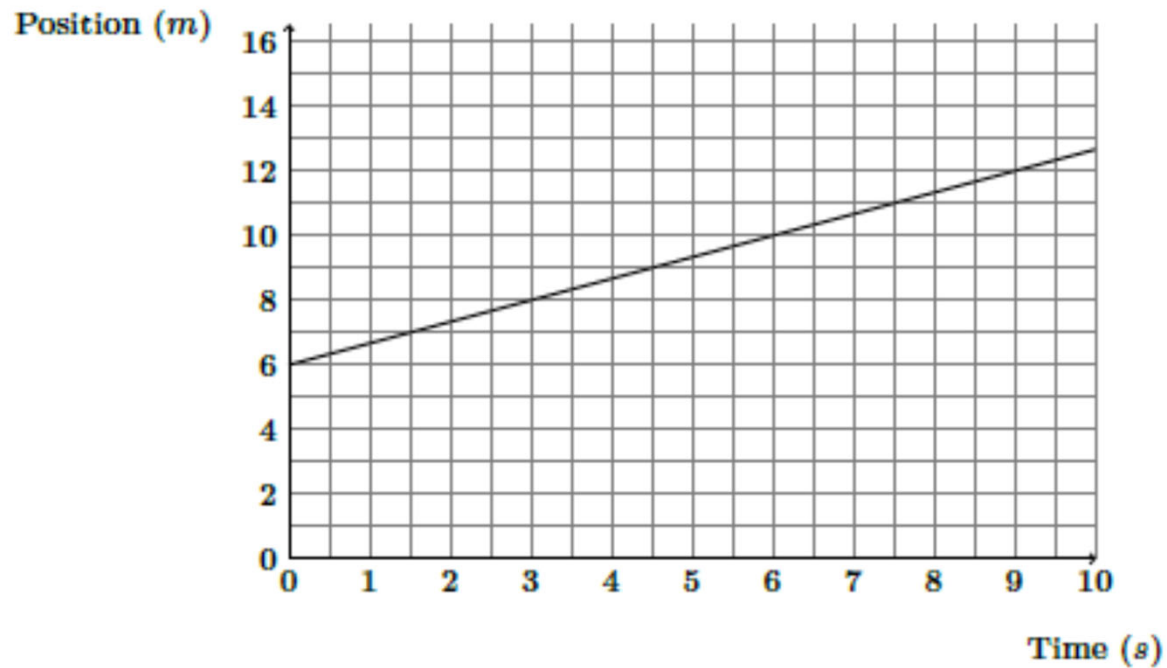
Find Slope of Graph

What is the slope of the graph below?



Correct-response rate: 30-60% ($N > 4000$)
(nearly independent of course or campus)

What is the slope of the graph below?



Most common error: Counting grid squares and ignoring numbers on axes

Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by $\approx 25\%$.

Algebra: Simultaneous Equations (Algebra-based course, ASU-T)

$$0.5y = 2x$$

$$78.4 - y = 8x$$

[Solve for x]

Numeric Version 61% correct ($N = 470$)

Algebra: Simultaneous Equations (Algebra-based course, ASU-T)

$$0.5y = 2x$$

$$78.4 - y = 8x$$

[Solve for x]

Numeric Version 61% correct ($N = 470$)

$$cy = dx$$

$$a - y = bx$$

[Solve for x]

Symbolic Version 31% correct ($N = 372$)

Algebra: Simultaneous Equations (Calculus-based course, ASU-T)

$$0.5y = 2x$$

$$78.4 - y = 8x$$

[Solve for x]

Numeric Version 79% correct ($N = 1205$)

Algebra: Simultaneous Equations (Calculus-based course, ASU-T)

$$\begin{aligned}0.5y &= 2x \\ 78.4 - y &= 8x\end{aligned}$$

[Solve for x]

Numeric Version 79% correct ($N = 1205$)

$$\begin{aligned}cy &= dx \\ a - y &= bx\end{aligned}$$

[Solve for x]

Symbolic Version 55% correct ($N = 1202$)

Symbolic

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

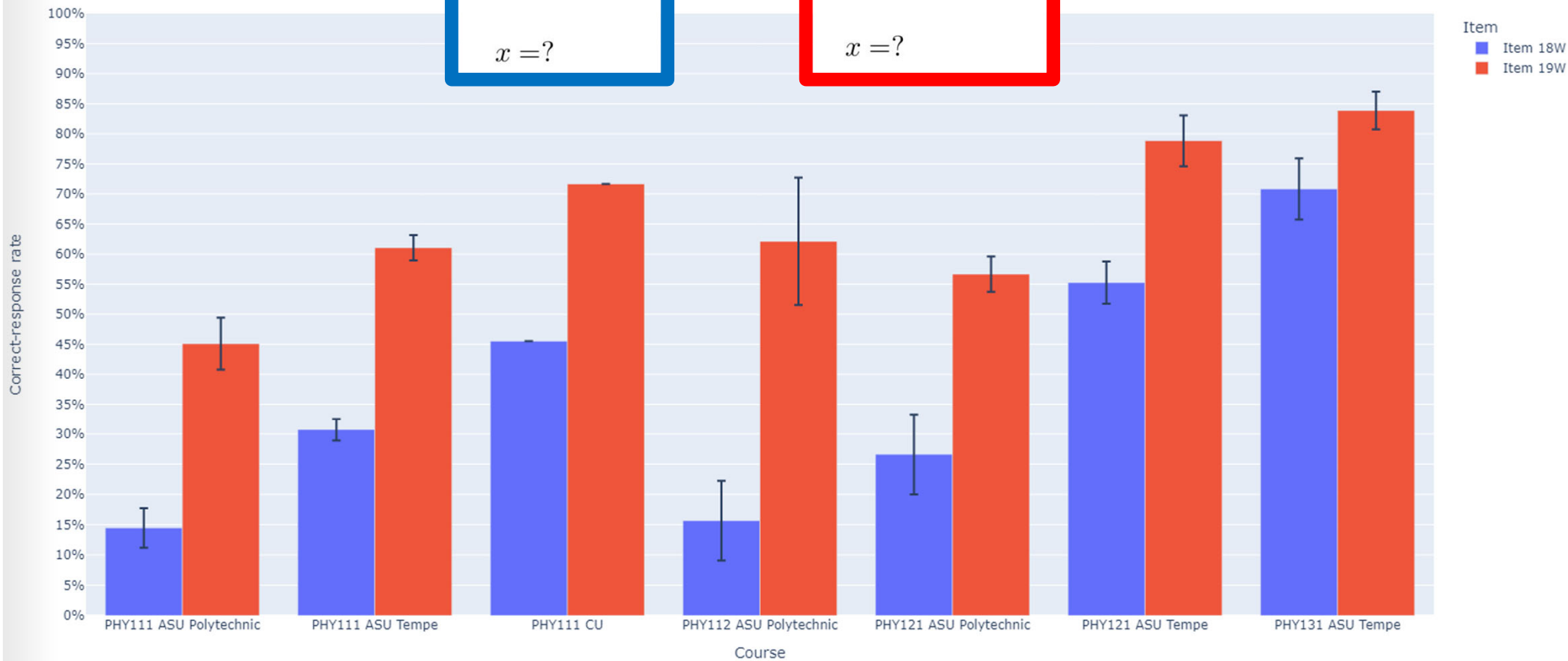
Numeric

$$0.5y = 2x$$

$$78.4 - y = 8x$$

$$x = ?$$

Course Averages



Confusion can result from the nature of the symbols themselves

Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Solve for x .

$$ax + b = cx + d$$

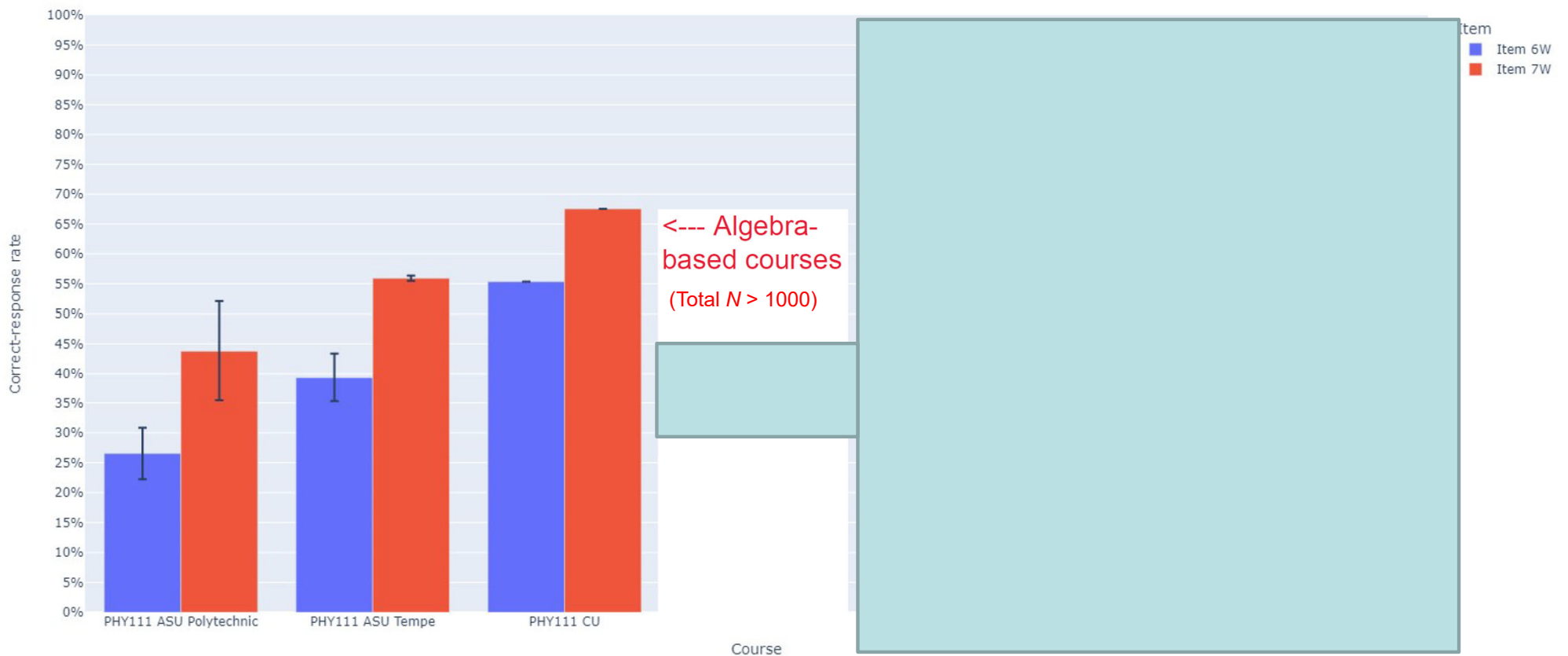
Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x .

$$ax + b = cx + d$$



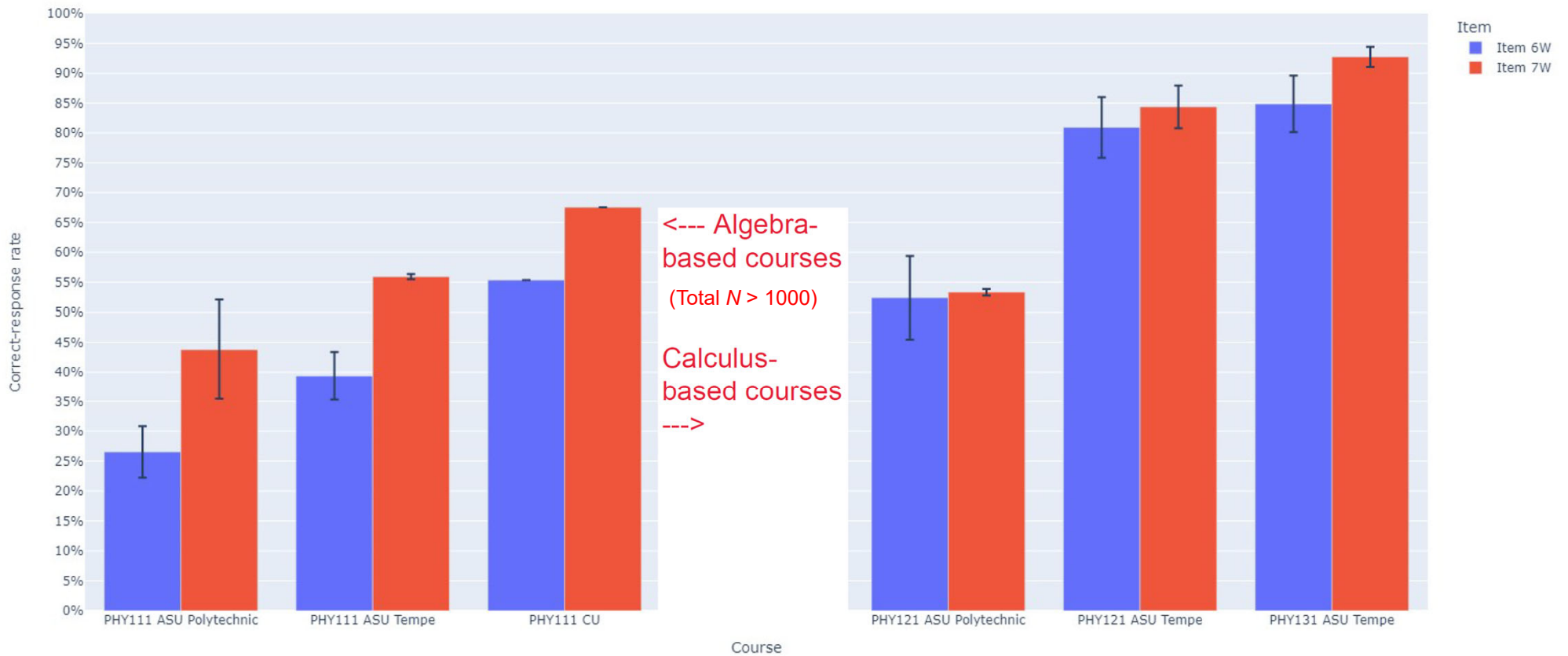
Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

Significantly lower correct-response rates on Greek-letter version in algebra-based courses

Solve for x .

$$ax + b = cx + d$$



Students favor non-standard solution methods

- Introductory physics students favor semi-arithmetic methods for solving algebraic equations; they do not “isolate the unknown variable.”

13. What is the numerical value of d ?

$$v^2 = v_0^2 + 2ad$$

13. What is the numerical value of d ?

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$$d = ?$$

(Please clearly *circle* your answer and show all work.)

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

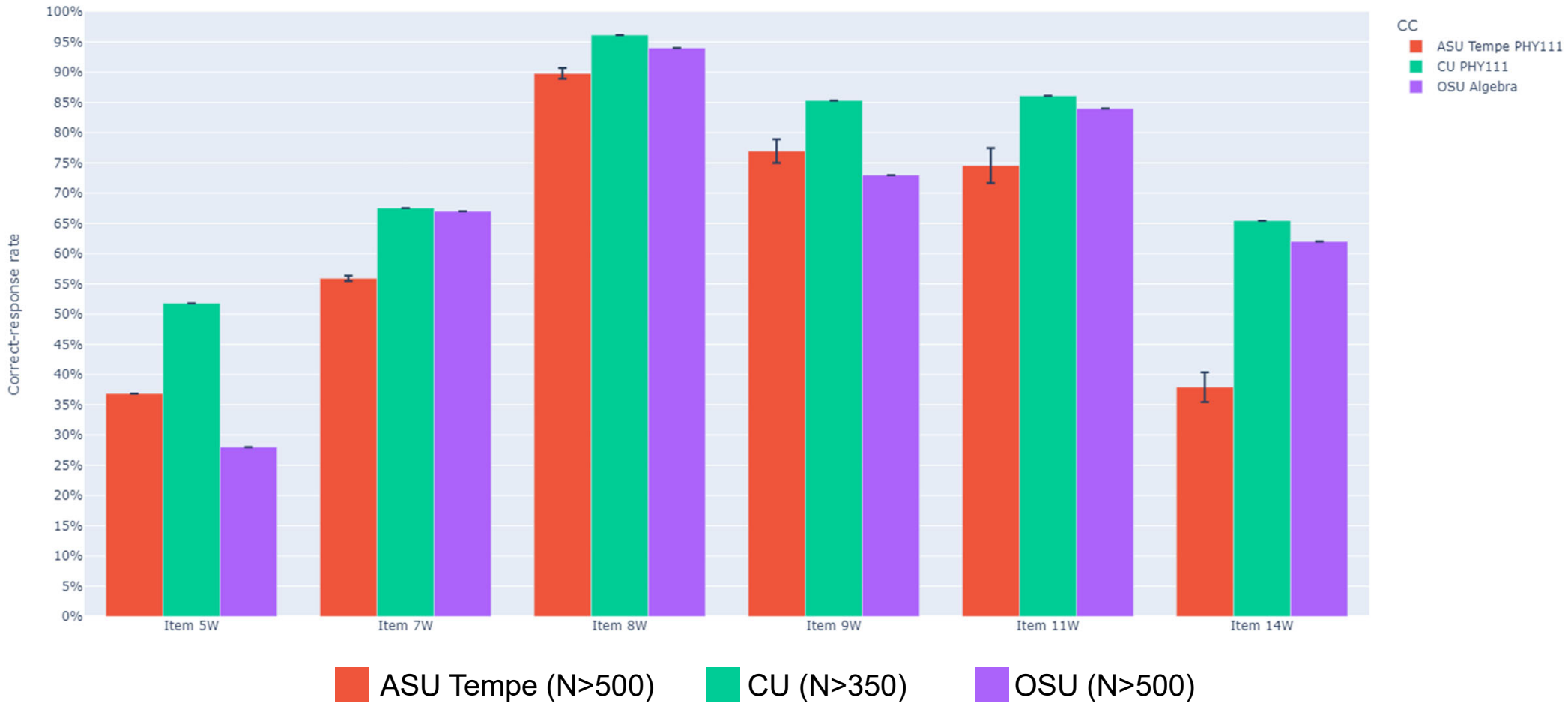
53/53 students solved it this way:

Handwritten student work on lined paper showing the derivation of $d = 60$:

$$v^2 = v_0^2 + 2ad$$
$$30^2 = (0)^2 + 2\left(\frac{60}{8}\right)d$$
$$900 = 0 + 2(7.5)d$$
$$900 = 15d$$
$$900/15 = d$$
$$60 = d$$

We observed these methods used on *thousands* of students' submissions

Results consistent among different universities



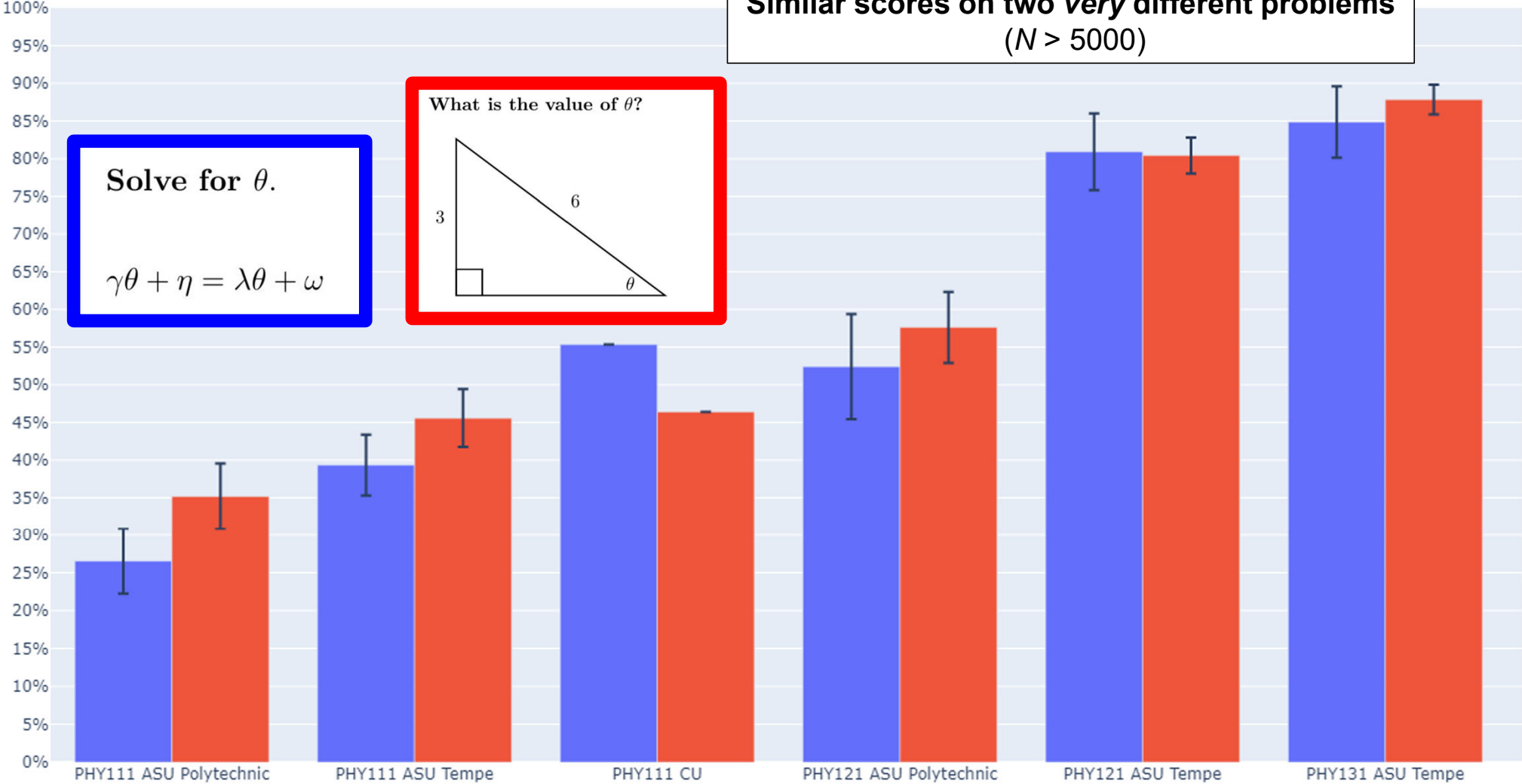
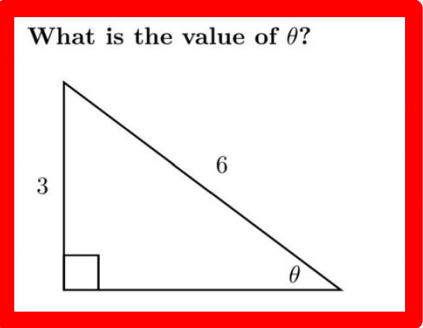
Correct-response rates: algebra-based course

Caution: Difficulties with one topic implies difficulties with others as well

- Students' scores on different problem types tend to track each other closely: relatively low scores on one type imply relatively low scores on the others
- Factor analysis shows only a single factor for entire diagnostic

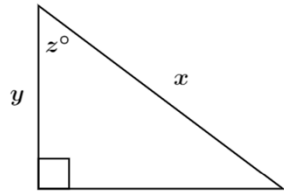
Similar scores on two very different problems
($N > 5000$)

Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$


On-line Version

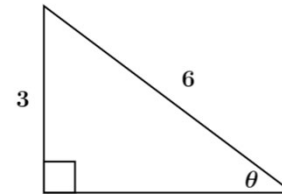
What is the length of side x ?



- A. $y \cos(z^\circ)$ D. $y / \cos(z^\circ)$ G. $\cos(z^\circ) / y$ J. $\sqrt{y^2 + z^2}$
 B. $y \sin(z^\circ)$ E. $y / \sin(z^\circ)$ H. $\sin(z^\circ) / y$ K. $\sqrt{z^2 - y^2}$
 C. $y \tan(z^\circ)$ F. $y / \tan(z^\circ)$ I. $\tan(z^\circ) / y$ L. y / z

(There may be more than one correct answer, but please select only ONE answer.)

What is the value of θ ?



- A. $\cos(3/6)$ D. $\cos^{-1}(3/6)$ G. 30° J. 27°
 B. $\sin(3/6)$ E. $\sin^{-1}(3/6)$ H. 45° K. $3/6$
 C. $\tan(3/6)$ F. $\tan^{-1}(3/6)$ I. 60° L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

$\cos(0^\circ) = ?$

- A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$\sin(90^\circ) = ?$

- A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$\tan(0^\circ) = ?$

- A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

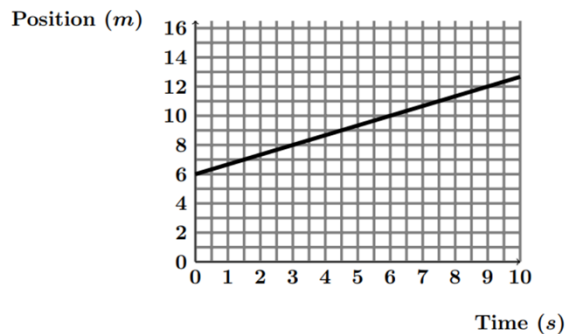
Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

- A. $\frac{\eta + \omega}{\gamma - \lambda}$ C. $\frac{\gamma - \lambda}{\omega - \eta}$ E. $\frac{\eta - \omega}{\gamma\lambda}$ G. $\frac{\omega - \eta}{\gamma - \lambda}$ I. $\frac{\eta - \omega + \gamma}{\lambda}$
 B. $\frac{\eta - \omega}{\lambda - \gamma}$ D. $\frac{\lambda - \gamma}{\eta - \omega}$ F. $\frac{\omega - \eta}{\gamma\lambda}$ H. $\frac{\omega - \eta}{\gamma + \lambda}$ J. $\frac{\omega - \eta + \lambda}{\gamma}$

(There may be more than one correct answer, but please select only ONE answer.)

What is the slope of the graph below?



- A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.
- B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.
- C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.
- D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

(There may be more than one correct answer, but please select only ONE answer.)

$$\frac{a/b}{c^2/d} = ?$$

- A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

$$\left(\frac{a}{3}\right)^3 = ?$$

- A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{a}{b}\right) = ?$$

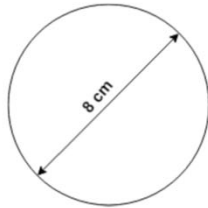
- A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{3}{4}\right) = ?$$

- A. $\frac{6}{8}$ B. $\frac{12}{8}$ C. $\frac{3}{8}$ D. $\frac{3}{2}$ E. $\frac{3}{4}$

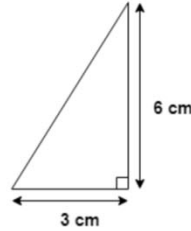
(There may be more than one correct answer, but please select only ONE answer.)



(a) Area of the circle = ?

- | | | |
|--------------------------|--------------------------|------------------------|
| A. $8\pi \text{ cm}^3$ | F. $8\pi \text{ cm}^2$ | K. $8\pi \text{ cm}$ |
| B. $16\pi \text{ cm}^3$ | G. $16\pi \text{ cm}^2$ | L. $16\pi \text{ cm}$ |
| C. $32\pi \text{ cm}^3$ | H. $32\pi \text{ cm}^2$ | M. $32\pi \text{ cm}$ |
| D. $64\pi \text{ cm}^3$ | I. $64\pi \text{ cm}^2$ | N. $64\pi \text{ cm}$ |
| E. $128\pi \text{ cm}^3$ | J. $128\pi \text{ cm}^2$ | O. $128\pi \text{ cm}$ |

(There may be more than one correct answer, but please select only ONE answer.)



(b) Area of the triangle = ?

- | | | |
|-----------------------|-----------------------|---------------------|
| A. 4.5 cm^3 | F. 4.5 cm^2 | K. 4.5 cm |
| B. 9 cm^3 | G. 9 cm^2 | L. 9 cm |
| C. 12 cm^3 | H. 12 cm^2 | M. 12 cm |
| D. 18 cm^3 | I. 18 cm^2 | N. 18 cm |
| E. 36 cm^3 | J. 36 cm^2 | O. 36 cm |

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

$$\frac{3}{2} = 7x$$

- A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

$$d = ?$$

- A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

(There may be more than one correct answer, but please select only ONE answer.)

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

- | | | | | |
|---------------------|----------------------|--------------------|------------------------------|--|
| A. $\frac{ac}{d+b}$ | C. $\frac{ac}{bc-d}$ | E. $\frac{ac}{db}$ | G. $\frac{a}{b+\frac{d}{c}}$ | I. $\frac{1}{b}\left(a-\frac{d}{c}\right)$ |
| B. $\frac{ac}{d-b}$ | D. $\frac{ac}{bc+d}$ | F. $\frac{a}{db}$ | H. $\frac{a}{b+d}$ | J. $\frac{c}{d}(a-b)$ |

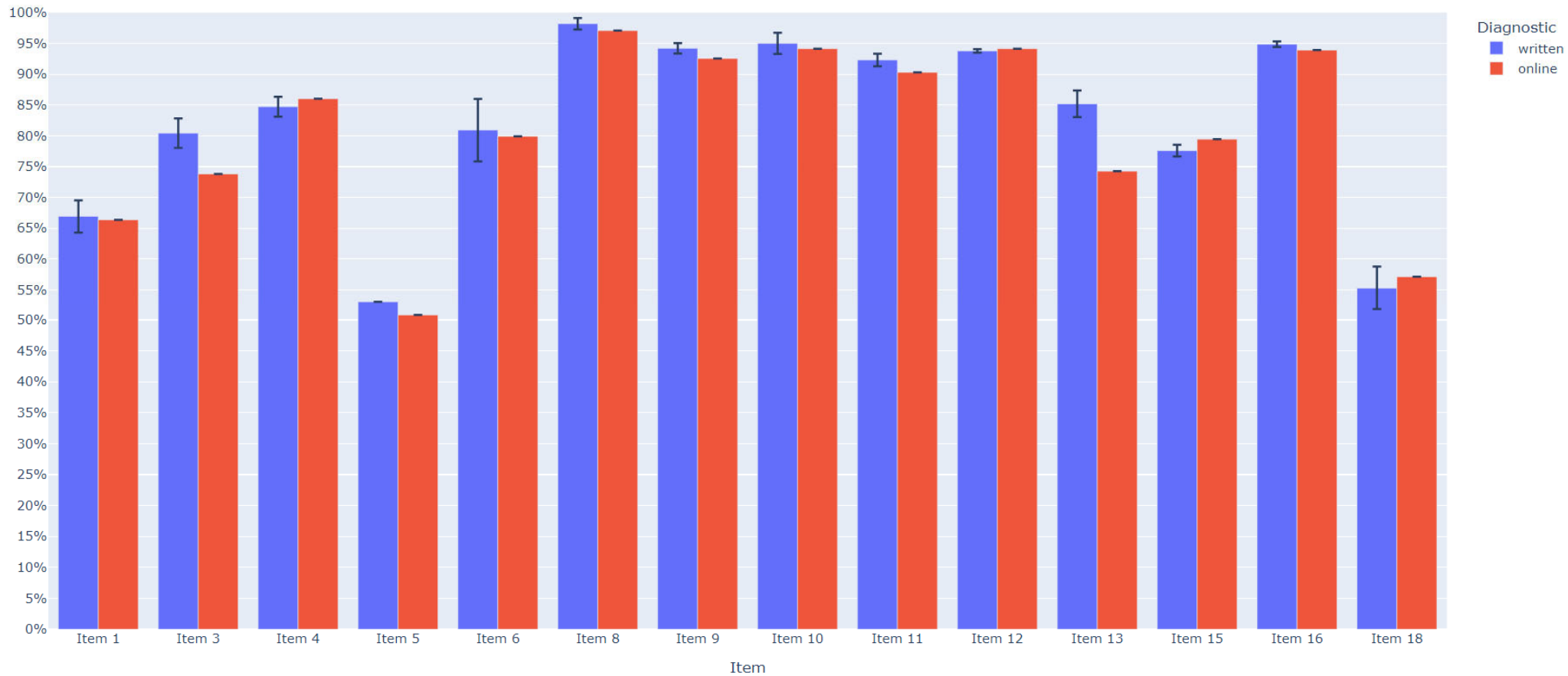
(There may be more than one correct answer, but please select only ONE answer.)

On-line and written versions yield consistent results

ASU Tempe PHY121 Averages

written

online



Findings from >80 Interviews: Students make many “careless” errors

- During interviews, students tended to self-correct approximately 60% of their initial errors with little or no prompting, suggesting that many errors are “careless.”
- These findings suggest that increased focus on improving students’ self-checking behavior might provide significant performance dividends.
 - However, studies have shown that making these improvements is quite challenging

Relation Between Scores and Grades

- Performance on **full online diagnostic** can *approximately* predict final course grade

Low Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade ≤ C+	Score ≥ 81% % grade ≤ C+	Score ≤ 57% % grade ≤ C+	Low-grade Ratio score ≤ 57% vs. score ≥ 81%

Low Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade \leq C+			
Alg-1 2021	ASU-P	78				

Alg-1: Algebra-based course, first semester

Low Course Grade vs. Full Diagnostic Score

Course	Campus	<i>N</i>	Overall % grade \leq C+			
Alg-1 2021	ASU-P	78	26%			

Alg-1: Algebra-based course, first semester

Low Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade \leq C+	Score \geq 81% % grade \leq C+	Score \leq 57% % grade \leq C+	Low-grade Ratio score \leq 57% vs. score \geq 81%
Alg-1 2021	ASU-P	78	26%	19%	38%	2.1

Alg-1: Algebra-based course, first semester

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Low Course Grade vs. Full Diagnostic Score

Course	Campus	<i>N</i>				
Alg-1 2021	ASU-P	78				
Alg-1 2022	ASU-P	93				
Alg-2	ASU-P	72				
Calc-1	UWF	103				

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

ASU-P: Arizona State University, Polytechnic campus

UWF: University of West Florida

Low Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade \leq C+			
Alg-1 2021	ASU-P	78	26%			
Alg-1 2022	ASU-P	93	19%			
Alg-2	ASU-P	72	29%			
Calc-1	UWF	103	39%			

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

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Low Course Grade vs. Full Diagnostic Score

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Alg-1 2021	ASU-P	78	26%			
Alg-1 2022	ASU-P	93	19%			
Alg-2	ASU-P	72	29%			
Calc-1	UWF	103	39%			

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

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Low Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade ≤ C+	Score ≥ 81% % grade ≤ C+	Score ≤ 57% % grade ≤ C+	Low-grade Ratio score ≤ 57% vs. score ≥ 81%
Alg-1 2021	ASU-P	78	26%	19%	38%	2.1
Alg-1 2022	ASU-P	93	19%	8%	28%	3.4
Alg-2	ASU-P	72	29%	14%	35%	2.6
Calc-1	UWF	103	39%	26%	54%	2.1

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

ASU-P: Arizona State University, Polytechnic campus

UWF: University of West Florida

*Students who scored low on math diagnostic pretest had **consistently** more “C” course grades than those who scored high*

High Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade \geq A-	Score \geq 81% % grade \geq A-	Score \leq 57% % grade \geq A-	High-grade Ratio score \geq 81% vs. score \leq 57%
Alg-1 2021	ASU-P	78				
Alg-1 2022	ASU-P	93				
Alg-2	ASU-P	72				
Alg-2	ASU-T	129				
Calc-1	UWF	103				
Calc-2	UWF	59				

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

ASU-T: Arizona State University, Tempe campus

UWF: University of West Florida

High Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade ≥ A-	Score ≥ 81% % grade ≥ A-	Score ≤ 57% % grade ≥ A-	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1 2021	ASU-P	78	35%			
Alg-1 2022	ASU-P	93	45%			
Alg-2	ASU-P	72	39%			
Alg-2	ASU-T	129	60%			
Calc-1	UWF	103	22%			
Calc-2	UWF	59	49%			

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

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High Course Grade vs. Full Diagnostic Score

Course	Campus	N	Overall % grade ≥ A-	Score ≥ 81% % grade ≥ A-	Score ≤ 57% % grade ≥ A-	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1 2021	ASU-P	78	35%	63%	15%	4.2
Alg-1 2022	ASU-P	93	45%	67%	28%	2.4
Alg-2	ASU-P	72	39%	64%	25%	2.6
Alg-2	ASU-T	129	60%	67%	55%	1.2
Calc-1	UWF	103	22%	40%	0%	"∞"
Calc-2	UWF	59	49%	61%	38%	1.6

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-1: Calculus-based course, first semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

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UWF: University of West Florida

*Students who scored high on math diagnostic pretest had **consistently more "A" course grades** than those who scored low*

Factors other than math preparation may influence course performance

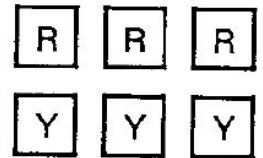
- For example:
 - Scientific reasoning skills, as measured by the Lawson Test of Scientific Reasoning
 - Physics concept knowledge, as measured by the Force Concept Inventory

Scientific reasoning skills: The 24-item Lawson test

Suppose you are given two clay balls of equal size and shape. The two clay balls also weigh the same. One ball is flattened into a pancake-shaped piece. *Which of these statements is correct?*

- a. The pancake-shaped piece weighs more than the ball
- b. The two pieces still weigh the same
- c. The ball weighs more than the pancake-shaped piece

Six square pieces of wood are put into a cloth bag and mixed about. The six pieces are identical in size and shape, however, three pieces are red and three are yellow. Suppose someone reaches into the bag (without looking) and pulls out one piece. *What are the chances that the piece is red?*



- a. 1 chance out of 6
- b. 1 chance out of 3
- c. 1 chance out of 2
- d. 1 chance out of 1
- e. cannot be determined

High Course Grade vs. Lawson Test of Scientific Reasoning Pretest Score

Course	Campus	N	Overall % grade \geq A-	Top-quartile Lawson % grade \geq A-	Bottom-quartile Lawson % grade \geq A-	High-grade Ratio Top quartile vs. Bottom quartile
Alg-1 2021	ASU-P	73	35%	65%	17%	3.9
Alg-1 2022	ASU-P	99	45%	62%	28%	2.2
Alg-2	ASU-P	73	39%	60%	15%	4.0
Alg-1	CU	469	22%	43%	6%	7.7
Calc-2	CU	276	25%	55%	9%	6.4
Alg-1 2007	LMU	24	42%	83%	0%	" ∞ "
Alg-1 2014	LMU	33	36%	88%	0%	" ∞ "
Alg-1 2018	LMU	47	38%	77%	17%	4.6
Alg-1 2021	LMU	27	48%	63%	0%	" ∞ "

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

CU: University of Colorado, Boulder

LMU: Loyola Marymount University

*Students who scored high on Lawson reasoning pretest had **consistently more "A"** course grades than those who scored low*

Low Course Grade vs. Lawson Test of Scientific Reasoning Pretest Score

Course	Campus	N	Overall % grade ≤ C+	Top-quartile Lawson % grade ≤ C+	Bottom-quartile Lawson % grade ≤ C+	Low-grade Ratio Bottom quartile vs. Top quartile
Alg-1 2021	ASU-P	73	26%	5%	56%	11.1
Alg-1 2022	ASU-P	99	19%	10%	28%	2.9
Alg-2	ASU-P	73	29%	10%	35%	3.5
Alg-1	CU	469	43%	21%	68%	3.2
Calc-2	CU	276	34%	13%	59%	4.5
Alg-1 2007	LMU	24	17%	0%	29%	"∞"
Alg-1 2014	LMU	33	24%	0%	67%	"∞"
Alg-1 2018	LMU	47	19%	15%	25%	1.6
Alg-1 2021	LMU	27	26%	13%	86%	6.9

Alg-1: Algebra-based course, first semester

Alg-2: Algebra-based course, second semester

Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus

CU: University of Colorado, Boulder

LMU: Loyola Marymount University

*Students who scored low on Lawson reasoning pretest had **consistently more "C"** course grades than those who scored high*

What Grade is Predicted by FCI Pretest Score?

- Henderson (2002), University of Minnesota ($N > 1000$)

FCI Pretest score: 0-30%

63-100%

A: 10%

A: 47%

Ratio: 4.7

C: 46%

C: 9%

Ratio: 5.1

Students who scored high on FCI pretest had higher course grades than those who scored low

What Grade is Predicted by FCI Pretest Score?

- Meltzer (2012/13), Arizona State University ($N > 100$)

FCI Pretest score: 0-30%

A: 12%

C: 26%

63-100%

A: 65%

C: 13%

Ratio: 5.4

Ratio: 2.0

Students who scored high on FCI pretest had higher course grades than those who scored low

What Grade is Predicted by FCI Pretest Score?

- Pollock & Dubson (2005), University of Colorado ($N=470$)

FCI Pretest score: 0-30%

A: 13%

C: 50%

63-100%

A: 59%

C: 19%

Ratio: 4.7

Ratio: 2.7

Students who scored high on FCI pretest had higher course grades than those who scored low

Factors are correlated, but not 100%

- Outliers using one prediction method can often be explained by high pretest scores on another predictor
- Students with uniformly low pretest scores can sometimes perform well with exceptional efforts in class

Summary

- Numerous factors influence students' physics course performance
- Previous preparation in calculational and reasoning skills is important, as well as physics concept knowledge, motivation, and effort