Pre-instruction Math Quiz May Predict Students' Physics Course Performance

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Acknowledgments

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Overview

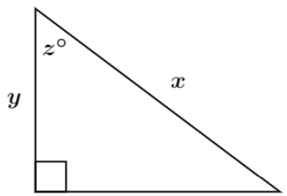
We have given diagnostic tests covering pre-college mathematics to over 7000 introductory physics students:

- Error rates were large enough to suggest that math difficulties can interfere with course performance;
- Results from five campuses at four different state universities were consistent with each other;
- Most data are from written free-response diagnostic; new online multiple-choice diagnostic has yielded results extremely similar to those of written version;
- Preliminary findings suggest that very high or low math pre-test scores may provide indications of ultimate physics course performance

All test items, multiple-choice version

(Calculators ***are*** allowed)

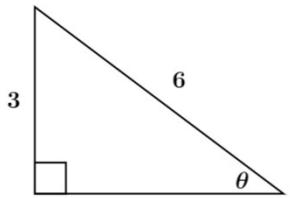
What is the length of side x?



A. $ycos(z°)$ D. $y/cos(z°)$ G. $cos(z°)/y$ J. $\sqrt{y^2+z^2}$
B. $ysin(z°)$ E. $y/sin(z°)$ H. $sin(z°)/y$ K. $\sqrt{z^2-y^2}$
C. $ytan(z°)$ F. $y/tan(z°)$ I. $tan(z°)/y$ L. y/z

(There may be more than one correct answer, but please select only ONE answer.)

What is the value of θ?



A. $cos(3/6)$ D. $cos^{-1}(3/6)$ G. 30° J. 27°
B. $sin(3/6)$ E. $sin^{-1}(3/6)$ H. 45° K. 3/6
C. $tan(3/6)$ F. $tan^{-1}(3/6)$ I. 60° L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

$cos(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$sin(90°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$tan(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

Solve for θ.

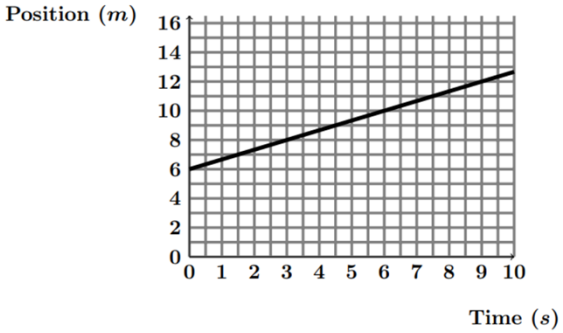
$$\gamma\theta + \eta = \lambda\theta + \omega$$

A. $\frac{\eta+\omega}{\gamma-\lambda}$ C. $\frac{\gamma-\lambda}{\omega-\eta}$ E. $\frac{\eta-\omega}{\gamma\lambda}$ G. $\frac{\omega-\eta}{\gamma-\lambda}$ I. $\frac{\eta-\omega+\gamma}{\lambda}$

B. $\frac{\eta-\omega}{\lambda-\gamma}$ D. $\frac{\lambda-\gamma}{\eta-\omega}$ F. $\frac{\omega-\eta}{\gamma\lambda}$ H. $\frac{\omega-\eta}{\gamma+\lambda}$ J. $\frac{\omega-\eta+\lambda}{\gamma}$

(There may be more than one correct answer, but please select only ONE answer.)

What is the slope of the graph below?



A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

(There may be more than one correct answer, but please select only ONE answer.)

$\left(\frac{a}{3}\right)^3 = ?$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(There may be more than one correct answer, but please select only ONE answer.)

$2\left(\frac{a}{b}\right) = ?$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

(There may be more than one correct answer, but please select only ONE answer.)

$\frac{a/b}{c^2/d} = ?$

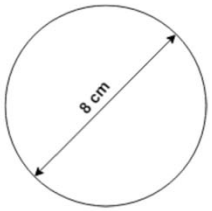
A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

$2\left(\frac{3}{4}\right) = ?$

A. $\frac{6}{8}$ B. $\frac{12}{8}$ C. $\frac{3}{8}$ D. $\frac{3}{2}$ E. $\frac{3}{4}$

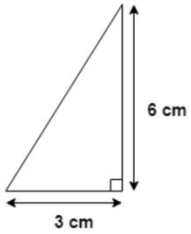
(There may be more than one correct answer, but please select only ONE answer.)



(a) Area of the circle = ?

A. 8π cm^3	F. 8π cm^2	K. 8π cm
B. 16π cm^3	G. 16π cm^2	L. 16π cm
C. 32π cm^3	H. 32π cm^2	M. 32π cm
D. 64π cm^3	I. 64π cm^2	N. 64π cm
E. 128π cm^3	J. 128π cm^2	O. 128π cm

(There may be more than one correct answer, but please select only ONE answer.)



(b) Area of the triangle = ?

A. 4.5 cm^3	F. 4.5 cm^2	K. 4.5 cm
B. 9 cm^3	G. 9 cm^2	L. 9 cm
C. 12 cm^3	H. 12 cm^2	M. 12 cm
D. 18 cm^3	I. 18 cm^2	N. 18 cm
E. 36 cm^3	J. 36 cm^2	O. 36 cm

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

$$\frac{3}{2} = 7x$$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

d =?

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

(There may be more than one correct answer, but please select only ONE answer.)

$$cy = dx$$

$$a - y = bx$$

x =?

A. $\frac{ac}{d+b}$ C. $\frac{ac}{bc-d}$ E. $\frac{ac}{db}$ G. $\frac{a}{b+\frac{d}{c}}$ I. $\frac{1}{b}\left(a-\frac{d}{c}\right)$

B. $\frac{ac}{d-b}$ D. $\frac{ac}{bc+d}$ F. $\frac{a}{db}$ H. $\frac{a}{b+d}$ J. $\frac{c}{d}\left(a-b\right)$

(There may be more than one correct answer, but please select only ONE answer.)

Key Finding: High error rates on many items

- Error rates of 30-60% appear consistently among diverse test items in all student populations.

Implication: Instructors may need to adjust expectations of students' operational abilities with trigonometry, graphing, algebra, etc.

Key Finding:
Even single test items are highly predictive

- Class-average scores on even a *single* diagnostic test item—regardless of which item was chosen—were highly predictive of average scores on all other diagnostic items covering varied topics.

Implication: It may be possible to diagnose the level of students' difficulties with only one or very few mathematics pretest items.

Predictability at Whole-Class Level

- Performance on **one single diagnostic item** can *accurately* predict class-average score on full diagnostic

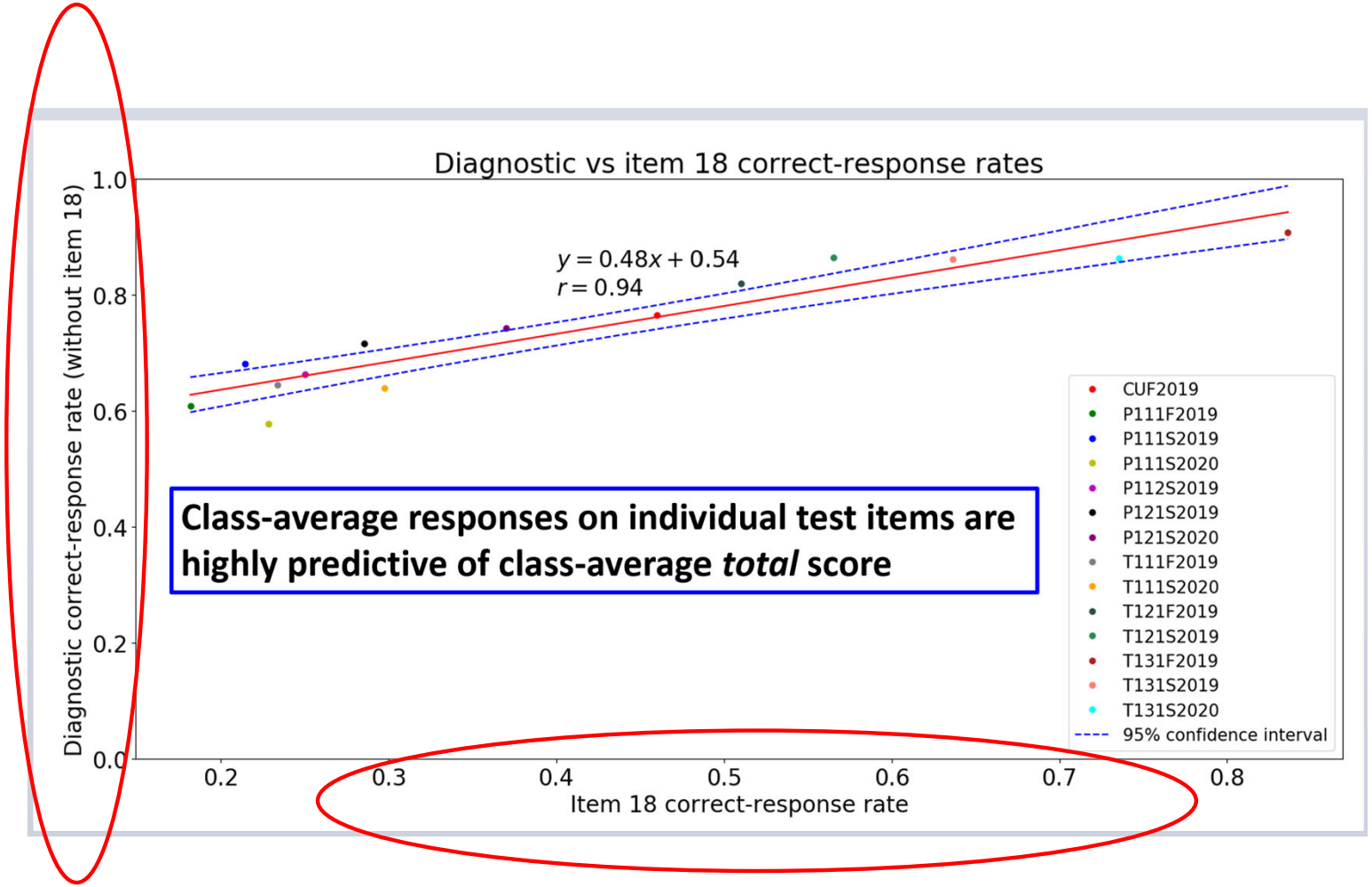
Example:

["#18"]

18. $cy = dx$

$a - y = bx$

$x = ?$


Diagnostic vs item 18 correct-response rates
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
$y = 0.48x + 0.54$
$r = 0.94$
Class-average responses on individual test items are highly predictive of class-average *total* score
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval

Tentative Finding: Math performance somewhat predictive of final grade

- Limited data: three class samples
- Clear pattern, but pattern type depends on student population
- No evidence of *causal* relationship

Predictability at Individual-Student Level

- Performance on **3-item subset** may *approximately* predict final course grade

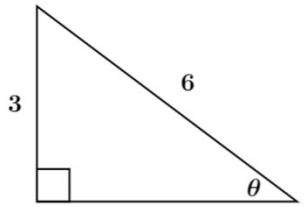
Example:

[#3, #11, #12]

[We found this to be the most predictive three-item set for the UWF student sample]

What is the value of θ?

#3



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B. $sin(3/6)$ E. $sin^{-1}(3/6)$ H. $45°$ K. $3/6$

C. $tan(3/6)$ F. $tan^{-1}(3/6)$ I. $60°$ L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

#12

$\frac{3}{2} = 7x$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$\frac{a/b}{c^2/d} = ?$

#11

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

Calculus-based Physics, 1st semester (UWF)

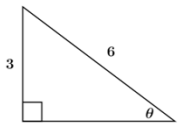
N = 95, 32% with final grade B+/A-/A

0 or 1 correct on [#3, #11, #12]	3/3 correct on [#3, #11, #12]
(*N* = 21)	(*N* = 44)
5% with final grade B+/A-/A	52% with final grade B+/A-/A

Three-item subset appears to be predictive of final grades…

What is the value of θ?

#3



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(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

#11

$\frac{3}{2} = 7x$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$\frac{a/b}{c^2/d} = ?$

#12

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

We found this set to be predictive of course grades for the UWF Spring 2021 sample (calculus-based course).

How would it perform in Fall 2021 in an algebra-based course at ASU Polytechic?

Algebra-based Physics, 1st semester (ASU Poly)

N = 82, 49% with final grade B+/A-/A

0 or 1 correct on [#3, #11, #12] (*N* = 20)	3/3 correct on [#3, #11, #12] (*N* = 20)
35% with final grade B+/A-/A	65% with final grade B+/A-/A

[We found that the very same three-item set was also predictive for the new ASU Poly sample]

Predictability at Individual-Student Level

- Performance on **full online diagnostic** can *approximately* predict final course grade

Previous data:

Calculus-based physics, 1st semester (UWF)

Algebra-based physics, 2nd semester (ASU Tempe)

New data:

Algebra-based physics, 1st semester (ASU Polytechnic)

Calculus-based Physics, 1st semester (UWF)

N = 95, 32% with final grade B+/A-/A

<70% correct responses (full diagnostic)

(*N* = 35)

6% with final grade B+/A-/A

>92% correct responses (full diagnostic)

(*N* = 21)

62% with final grade B+/A-/A

Algebra-based Physics, 2nd semester (ASU Tempe)

N = 118, 59% with final grade A-/A/A+

<86% correct responses (full diagnostic)

(*N* = 101)

53% with final grade A-/A/A+

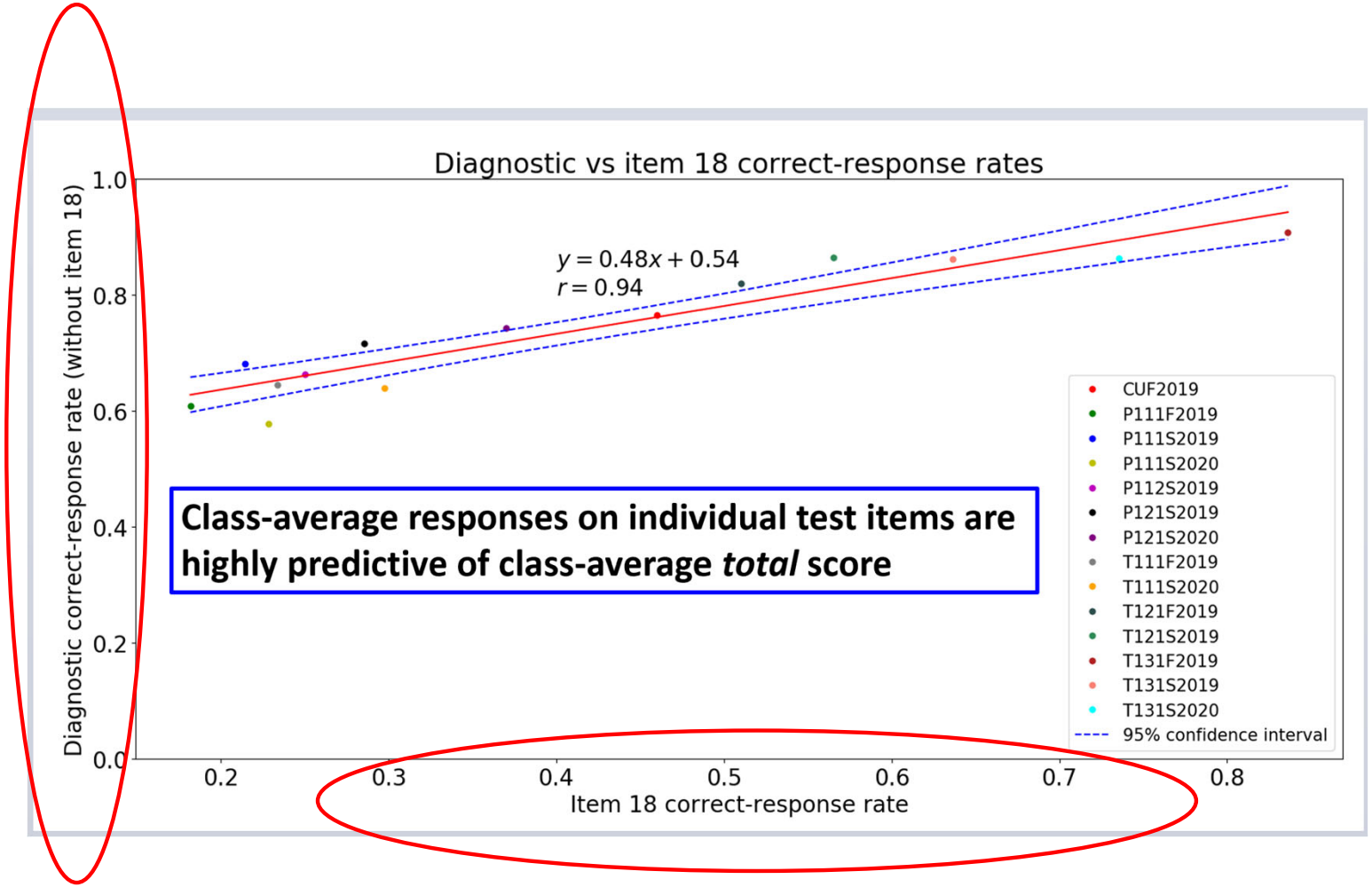
>92% correct responses (full diagnostic)

(*N* = 17)

94% with final grade A-/A/A+

New Data: Fall 2021, ASU Polytechnic

- Algebra-based course, 1st semester (mechanics)
- Very light use of mathematics: little trigonometry, single-variable equations only; limited use of symbols; almost all graphing was qualitative (no numbers or units)
- Taught by DEM; two sections taught back-to-back; samples combined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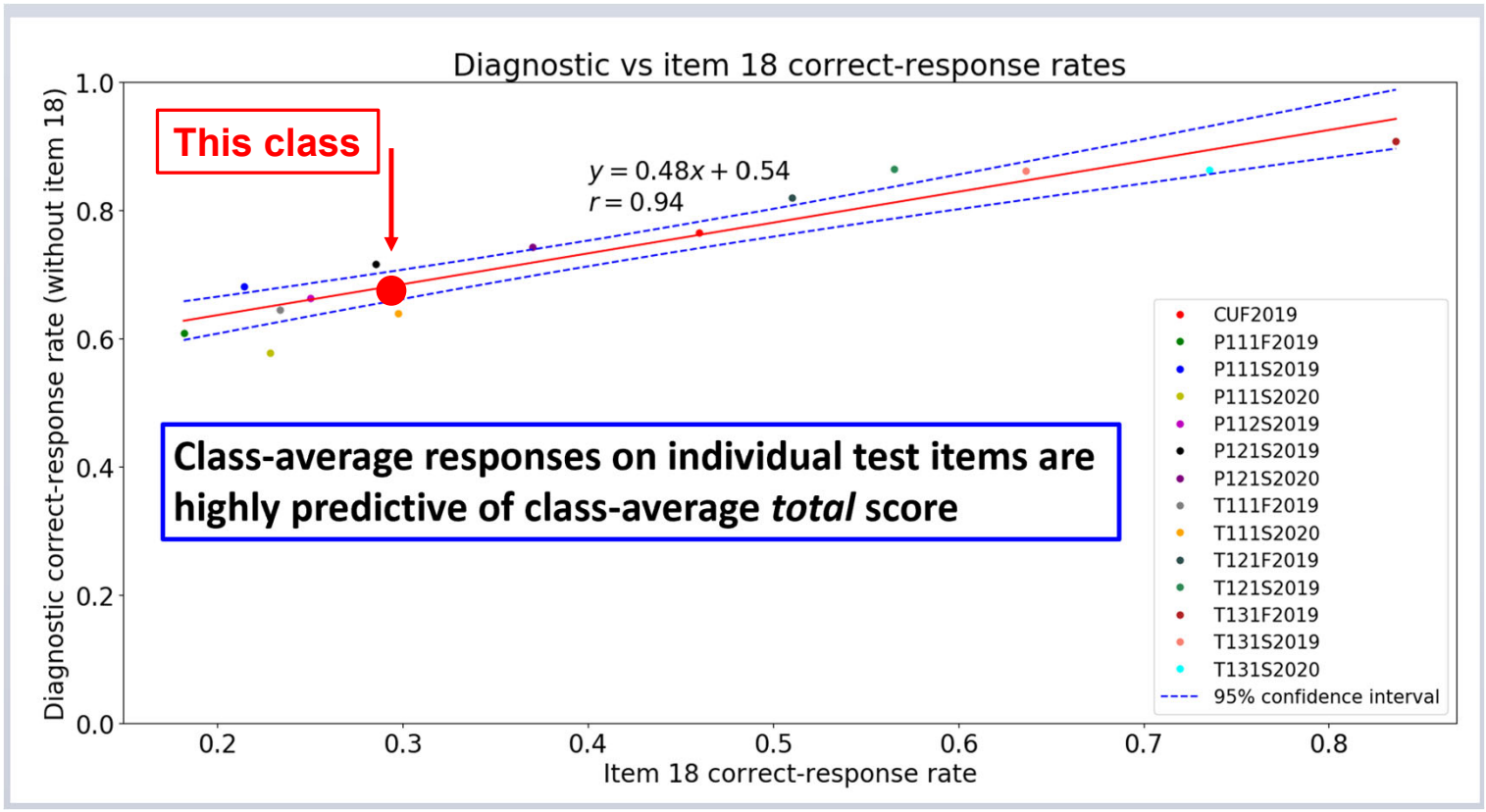
T131F2019
T131S2019
T131S2020
95% confidence interval


Diagnostic vs item 18 correct-response rates
This class
$y = 0.48x + 0.54$
$r = 0.94$
Class-average responses on individual test items are highly predictive of class-average *total* score
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval

Algebra-based Physics, 1st semester (ASU Poly)

N = 82, 34% with final grade A-/A/A+

<57% correct responses (full diagnostic)

(*N* = 29)

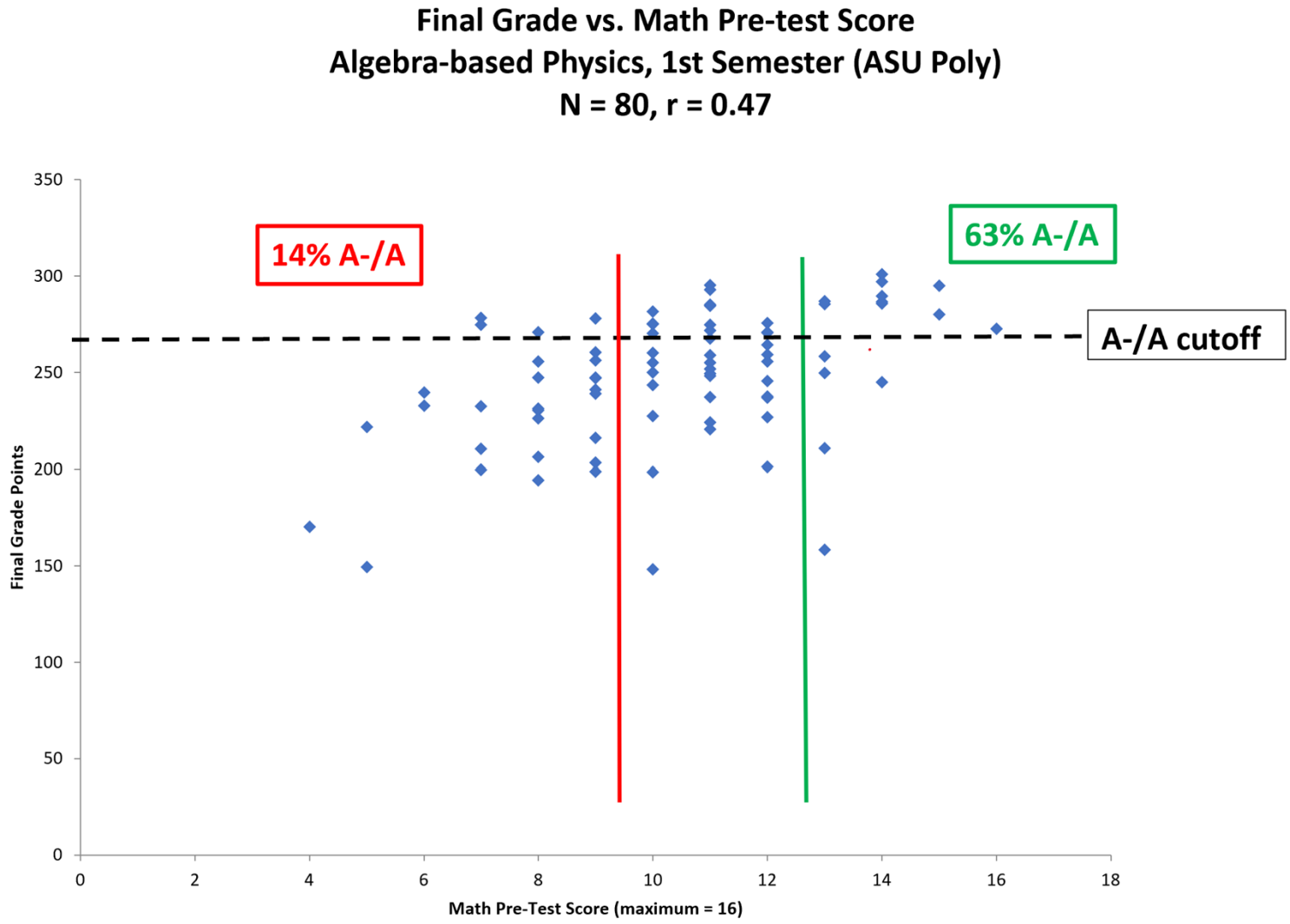
14% with final grade A-/A/A+

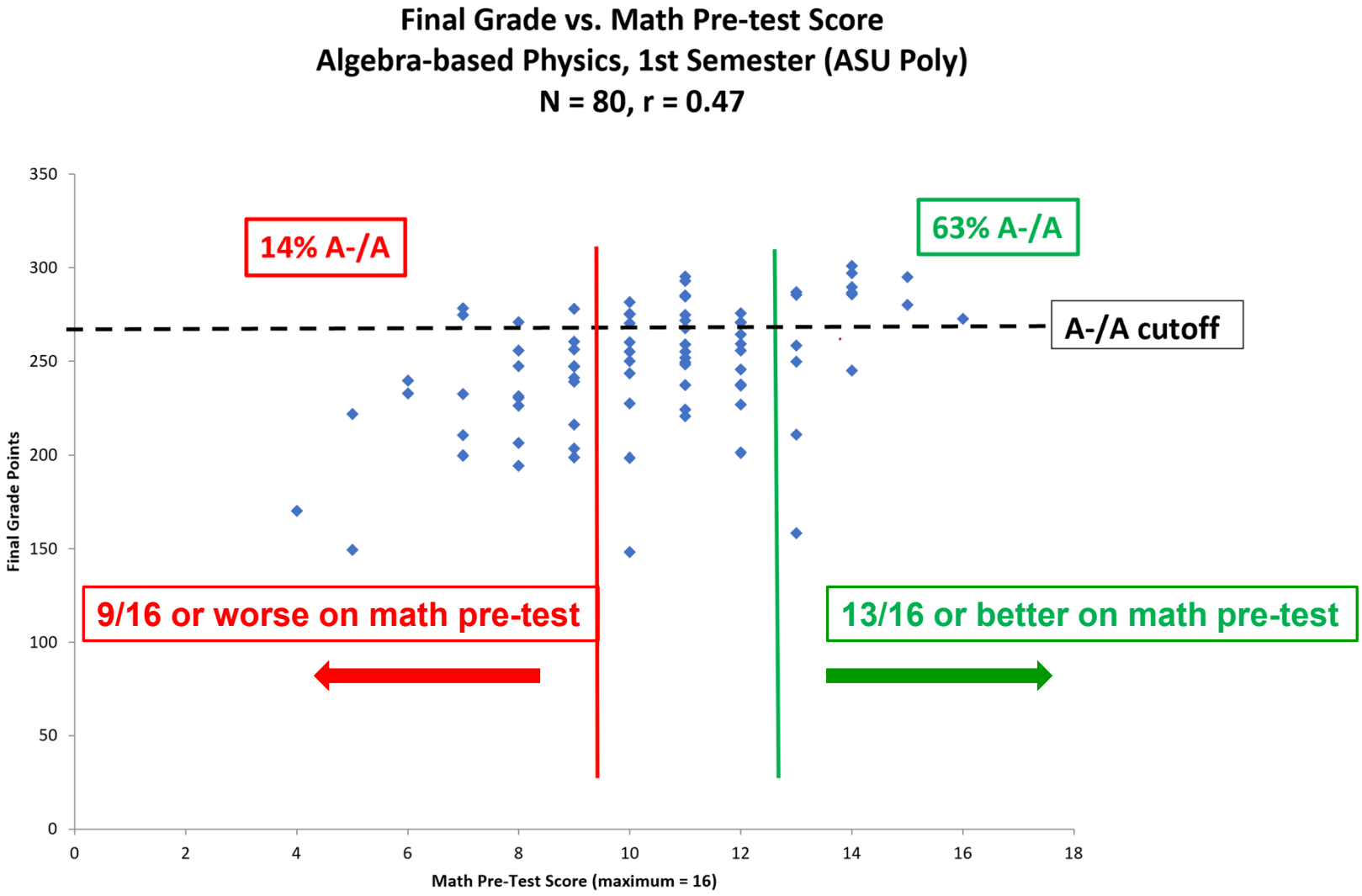
>81% correct responses (full diagnostic)

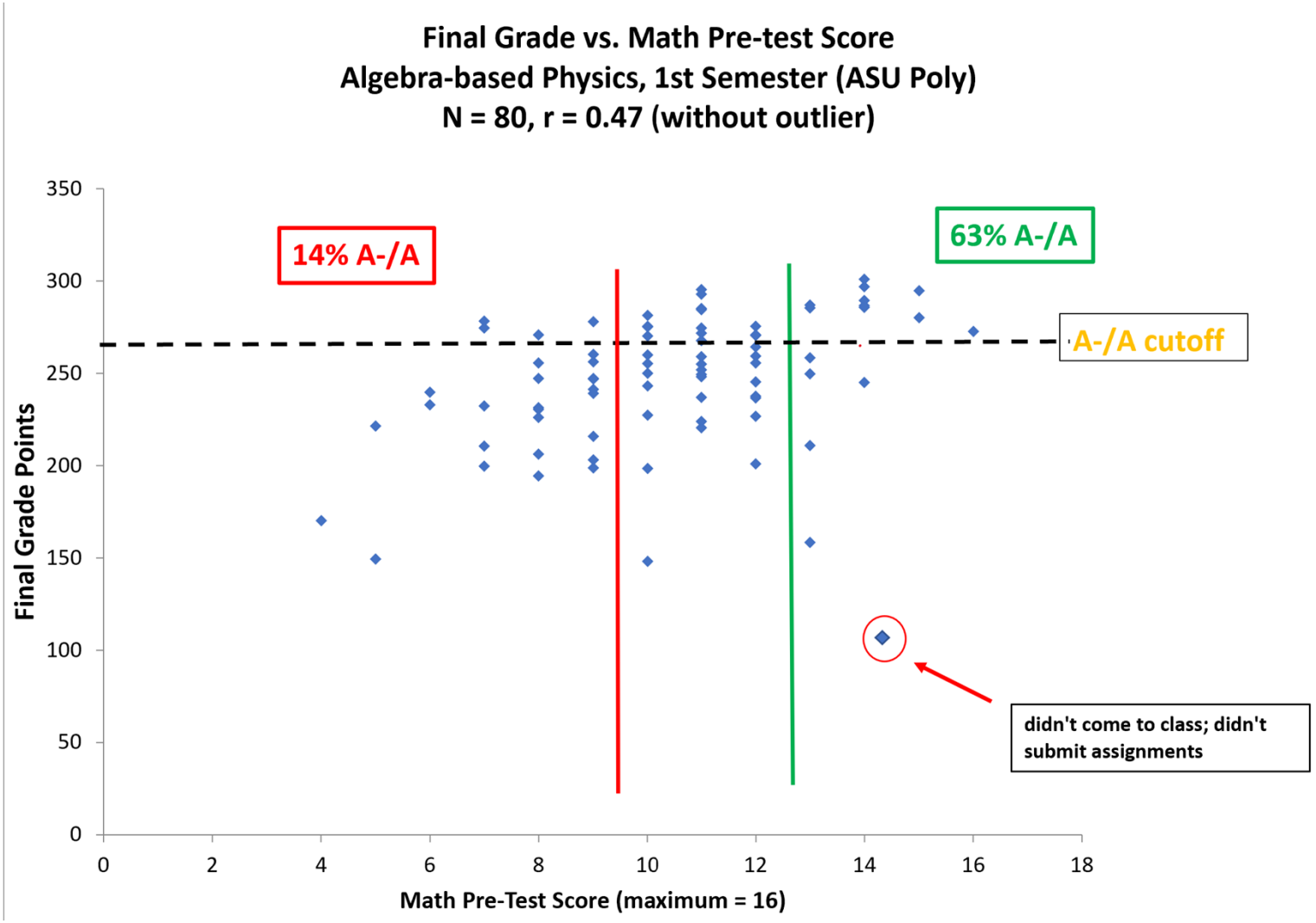
(*N* = 16)

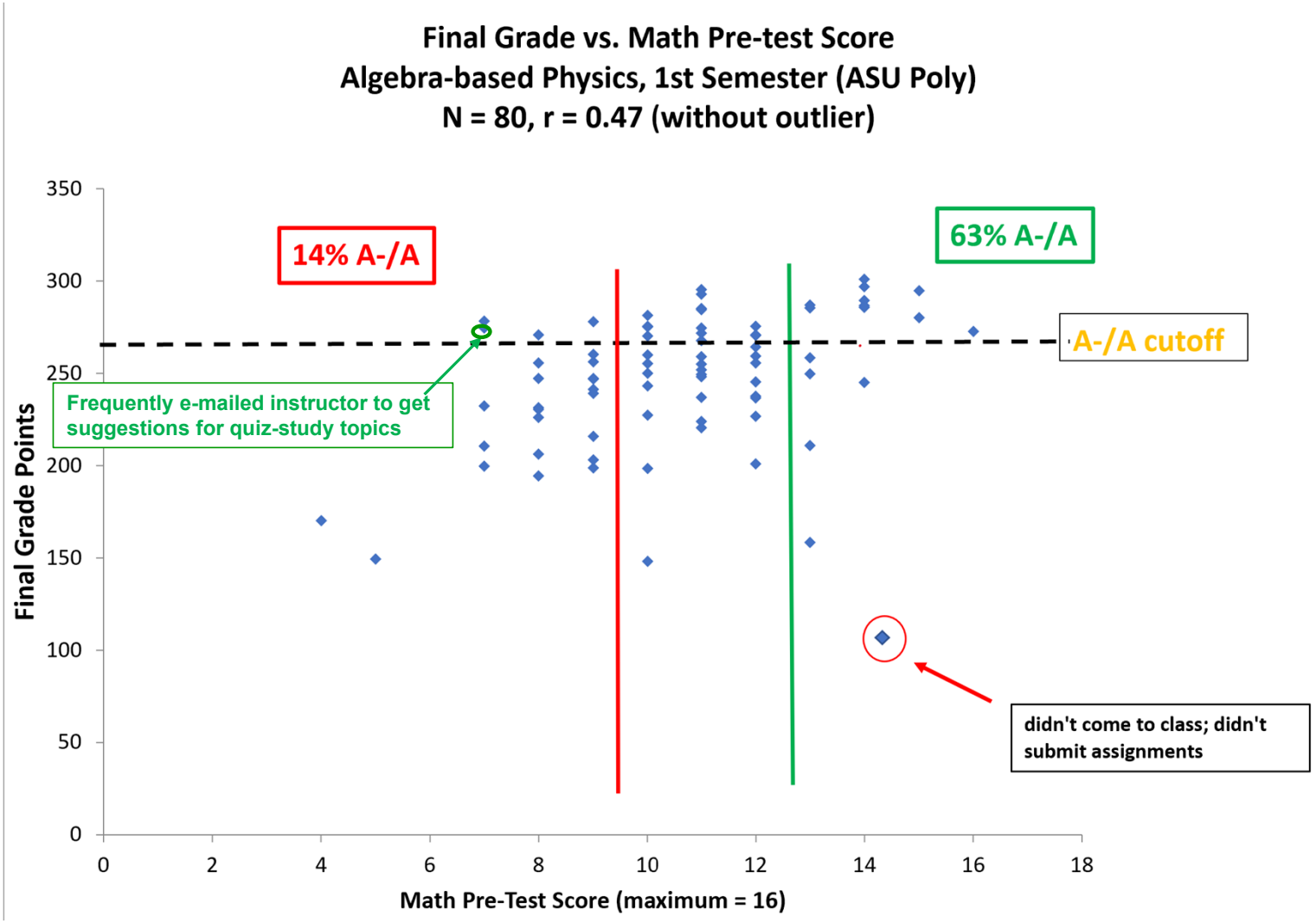
63% with final grade A-/A/A+

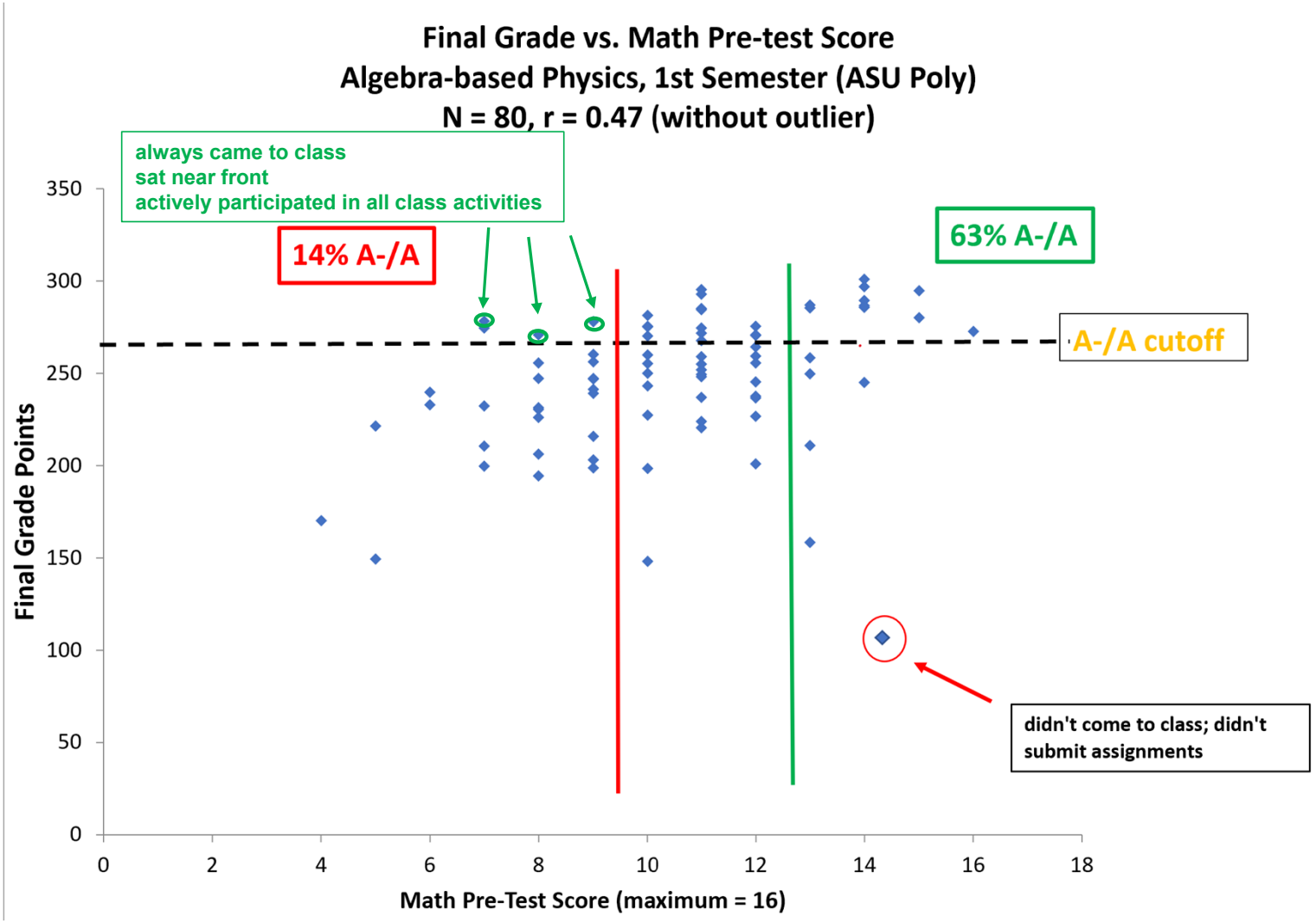
But here, we can examine individual data points [students] in more detail…

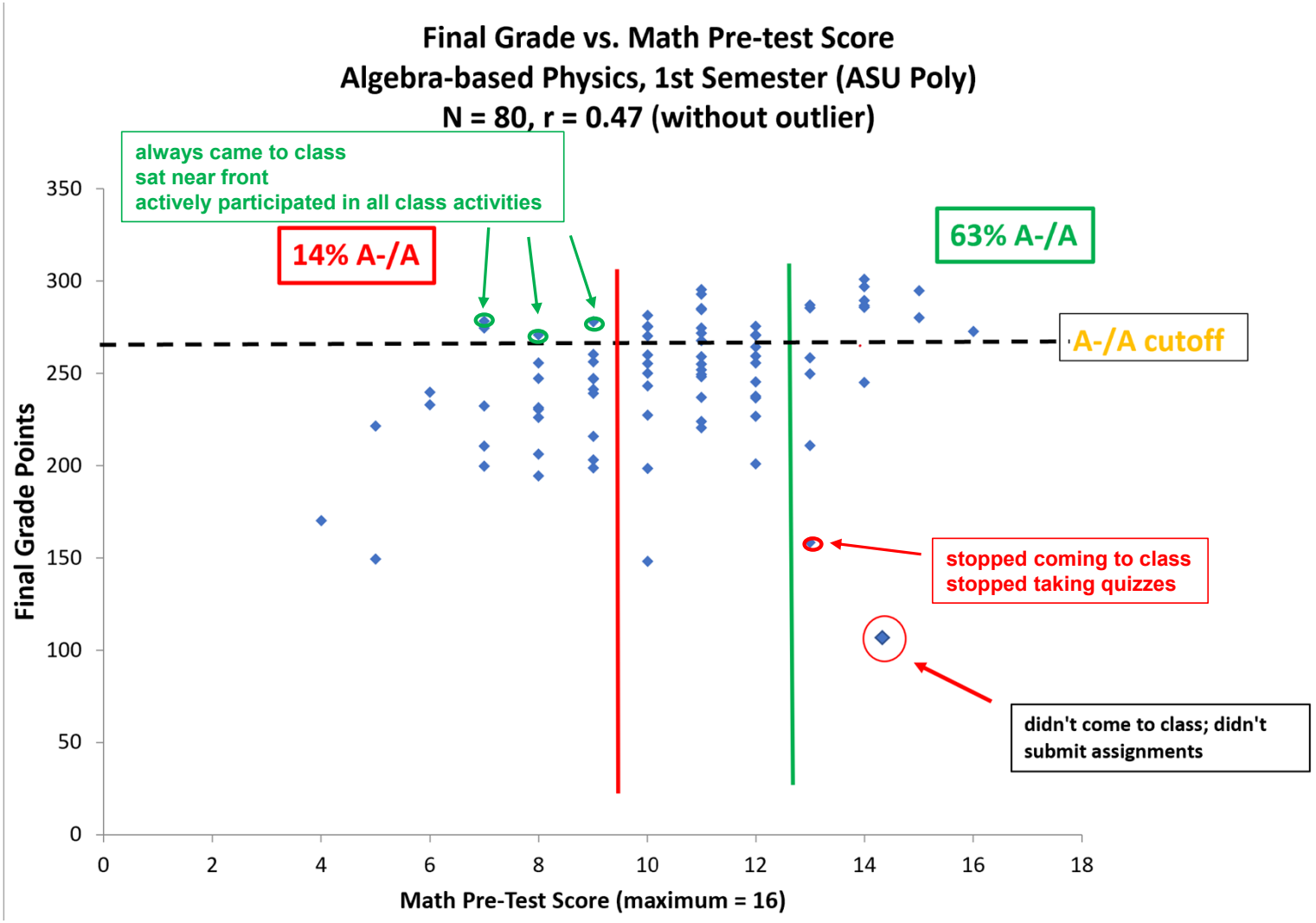

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47
14% A-/A
63% A-/A
A-/A cutoff
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

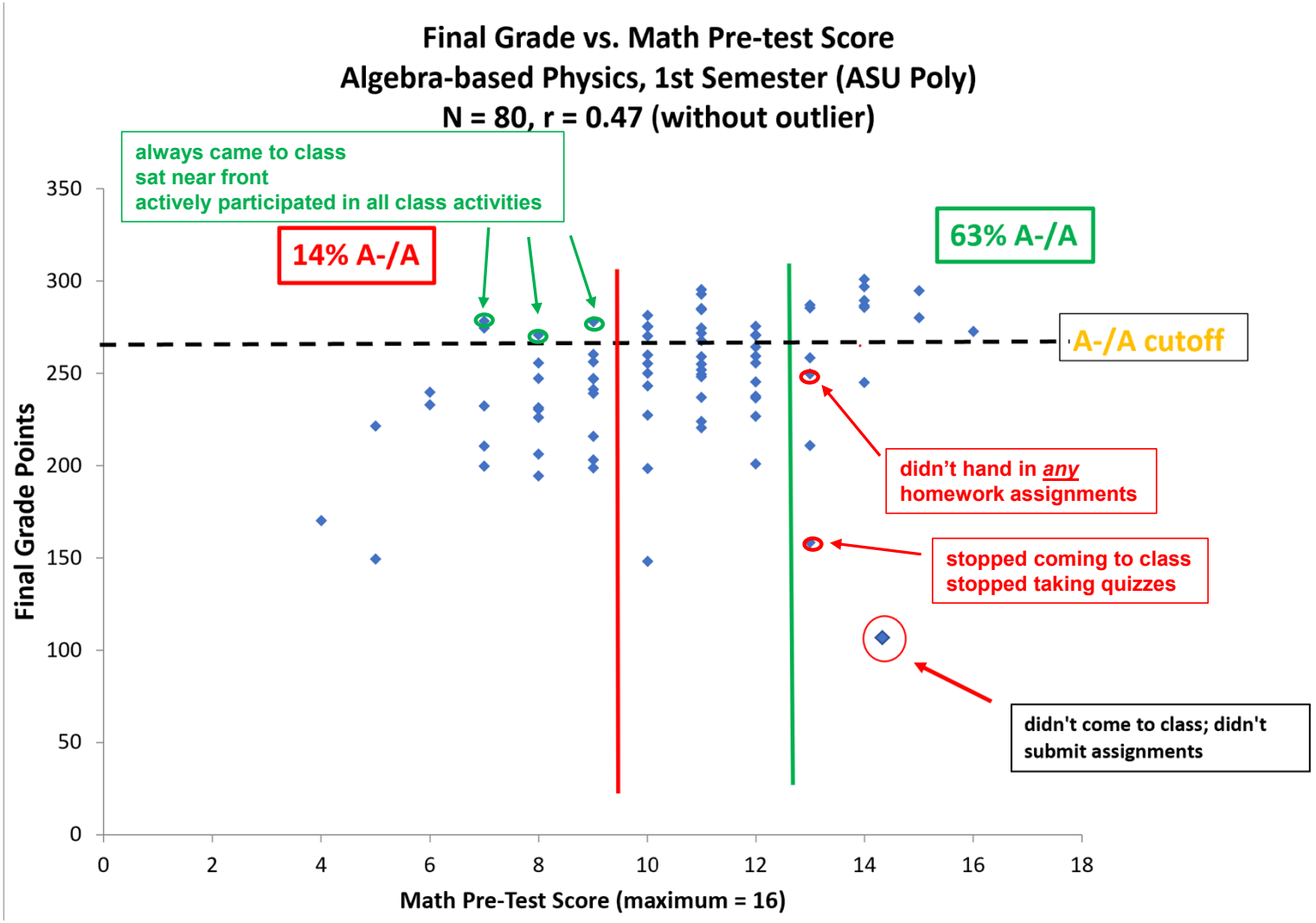

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47
14% A-/A
63% A-/A
A-/A cutoff
9/16 or worse on math pre-test
13/16 or better on math pre-test
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

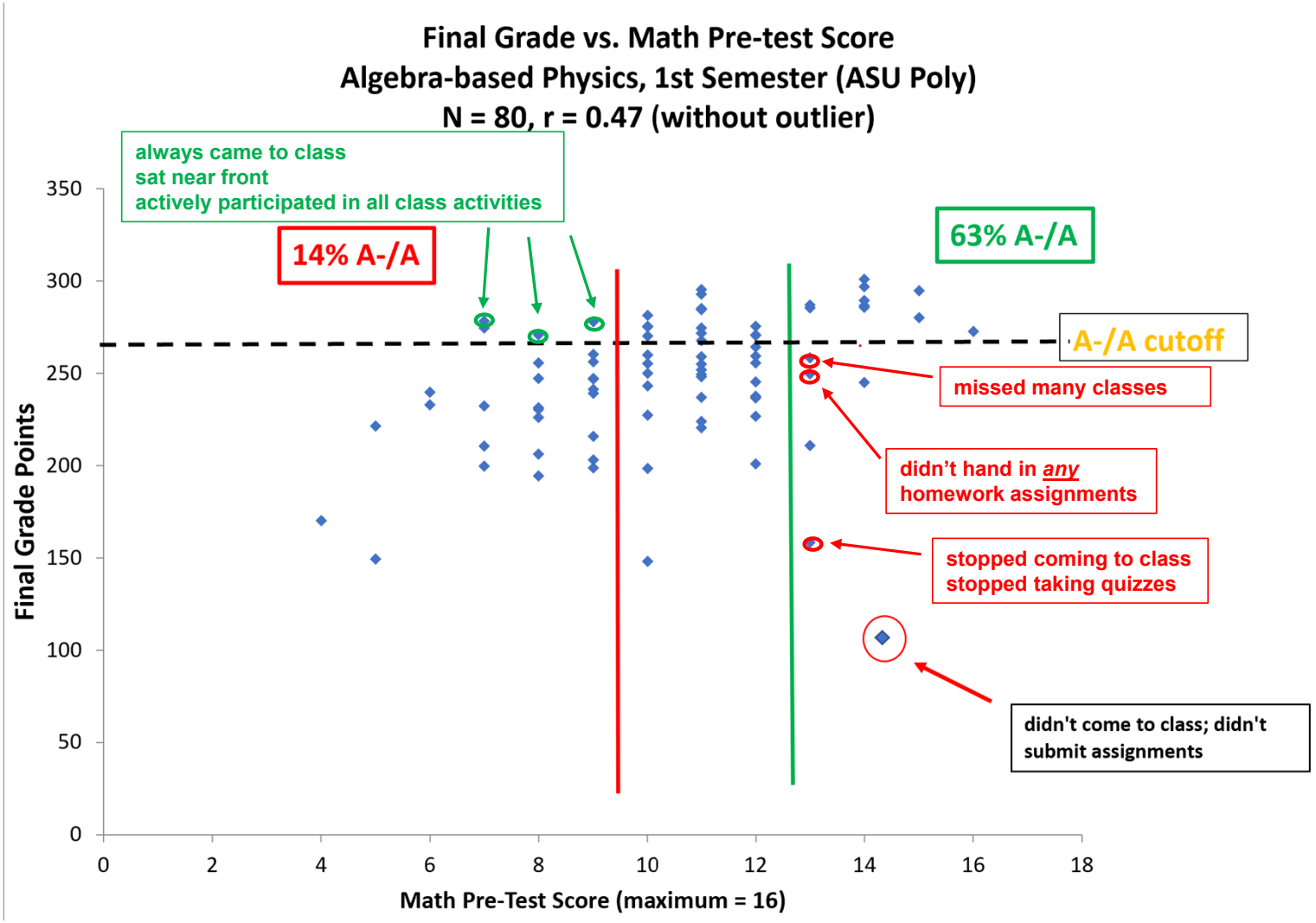

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
14% A-/A
63% A-/A
A-/A cutoff
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

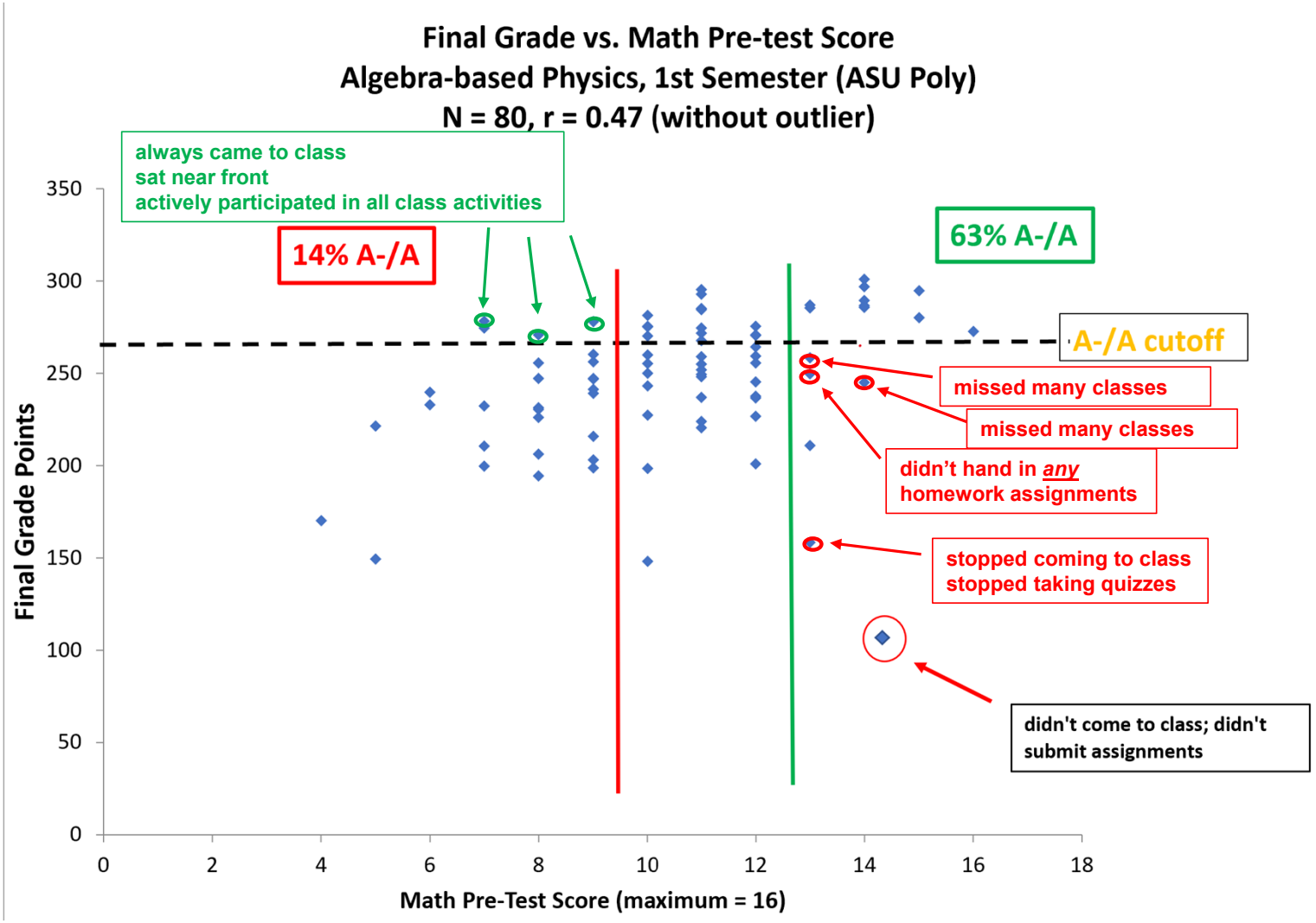

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
14% A-/A
63% A-/A
A-/A cutoff
Frequently e-mailed instructor to get suggestions for quiz-study topics
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

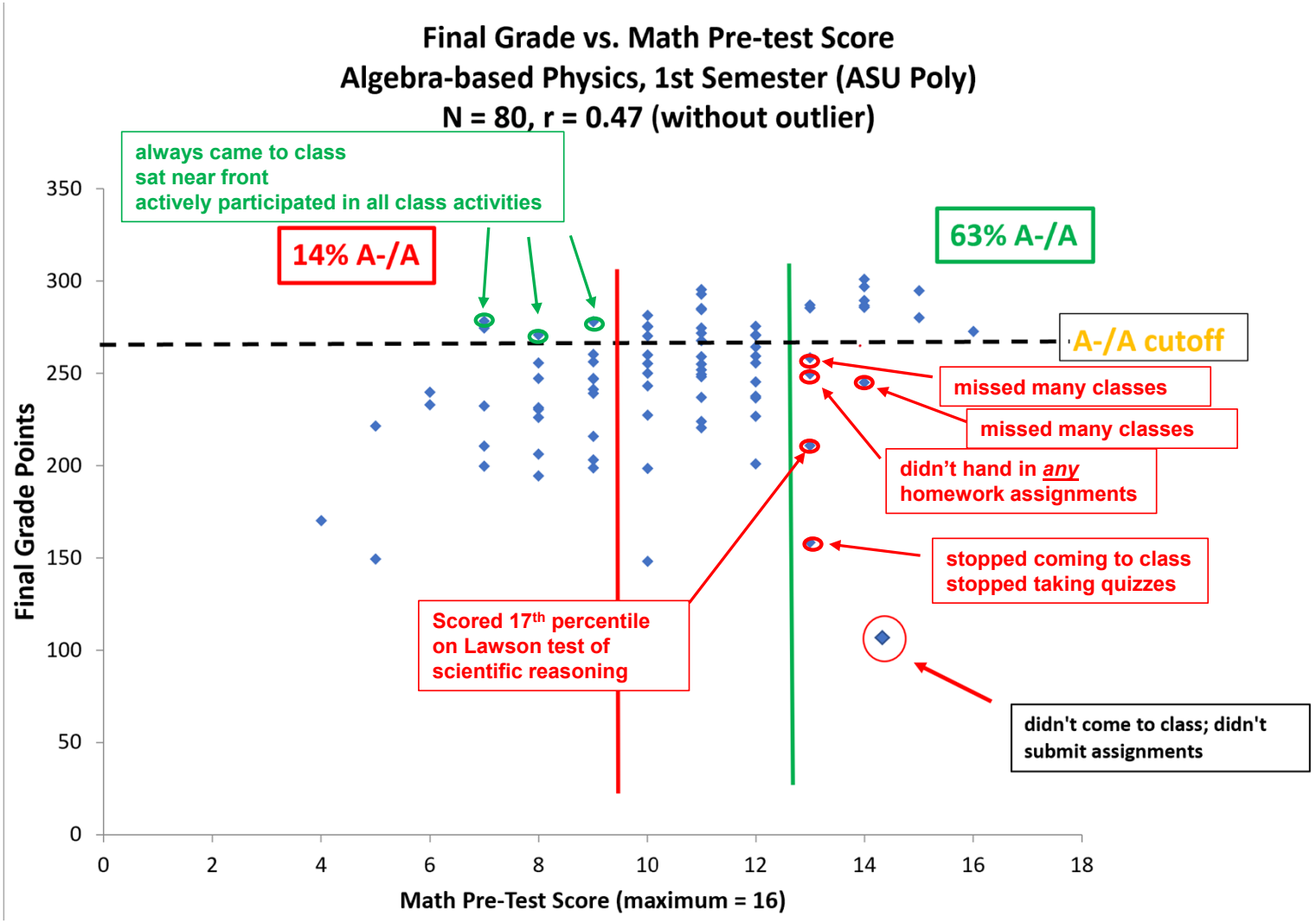

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

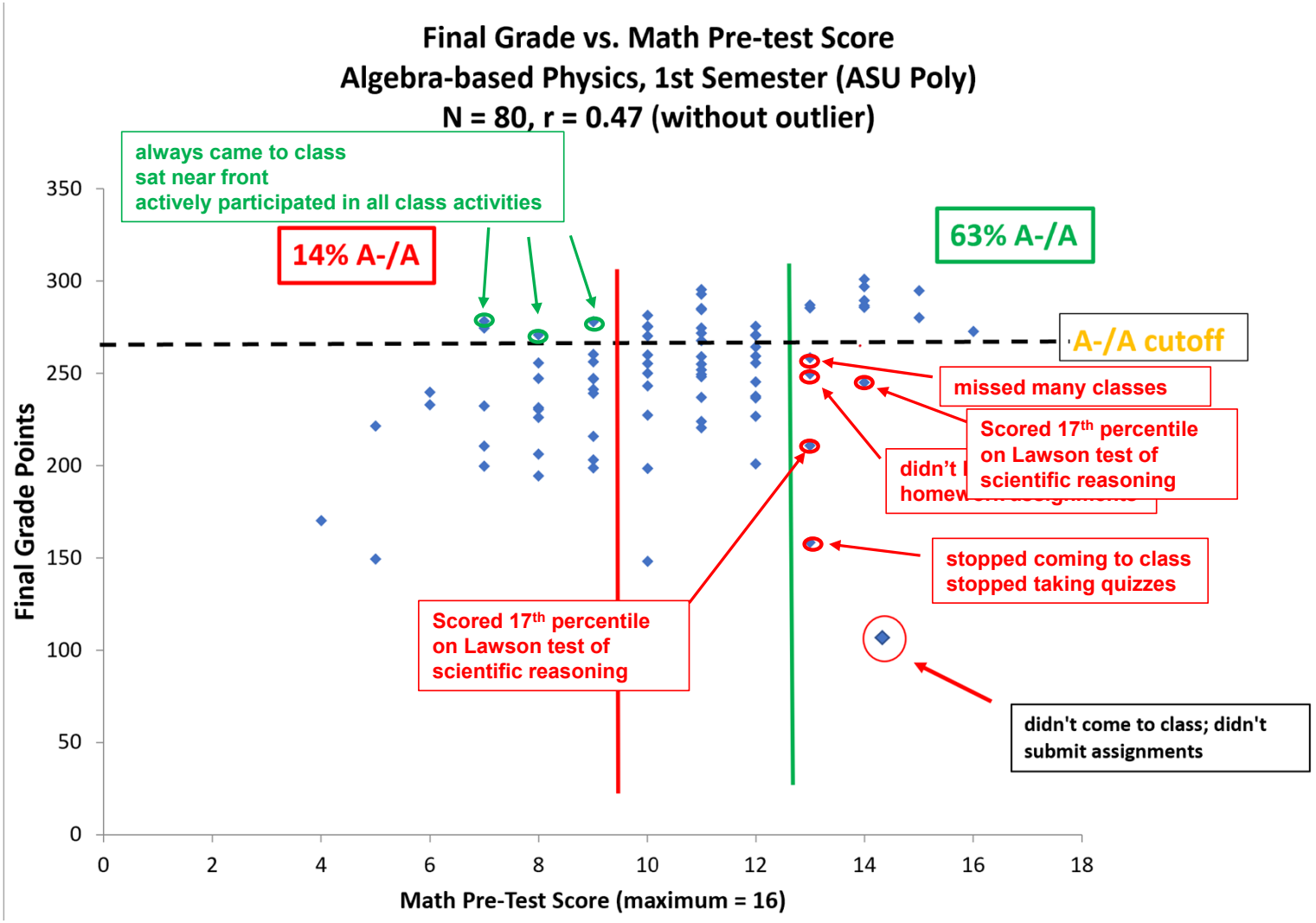

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

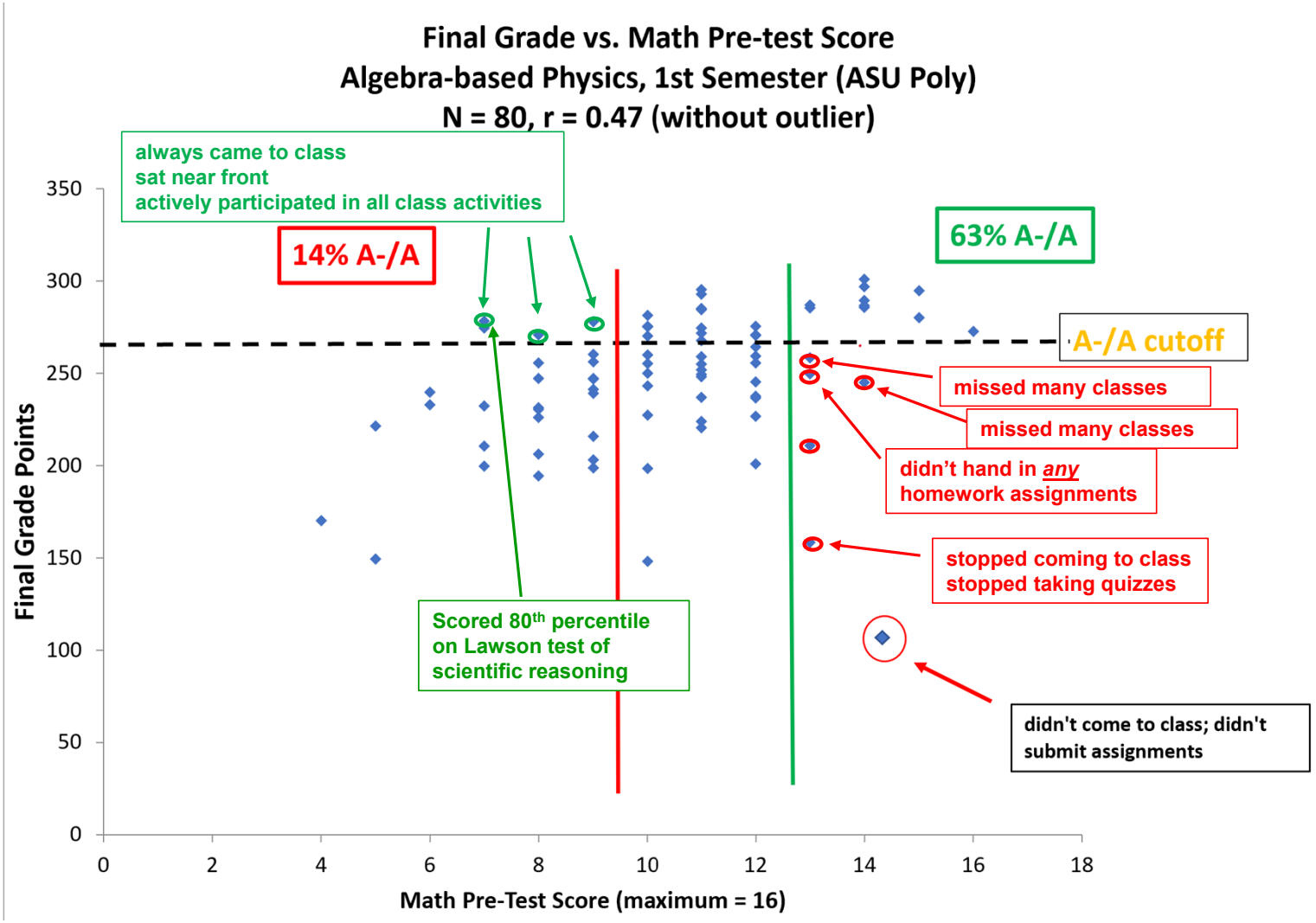

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

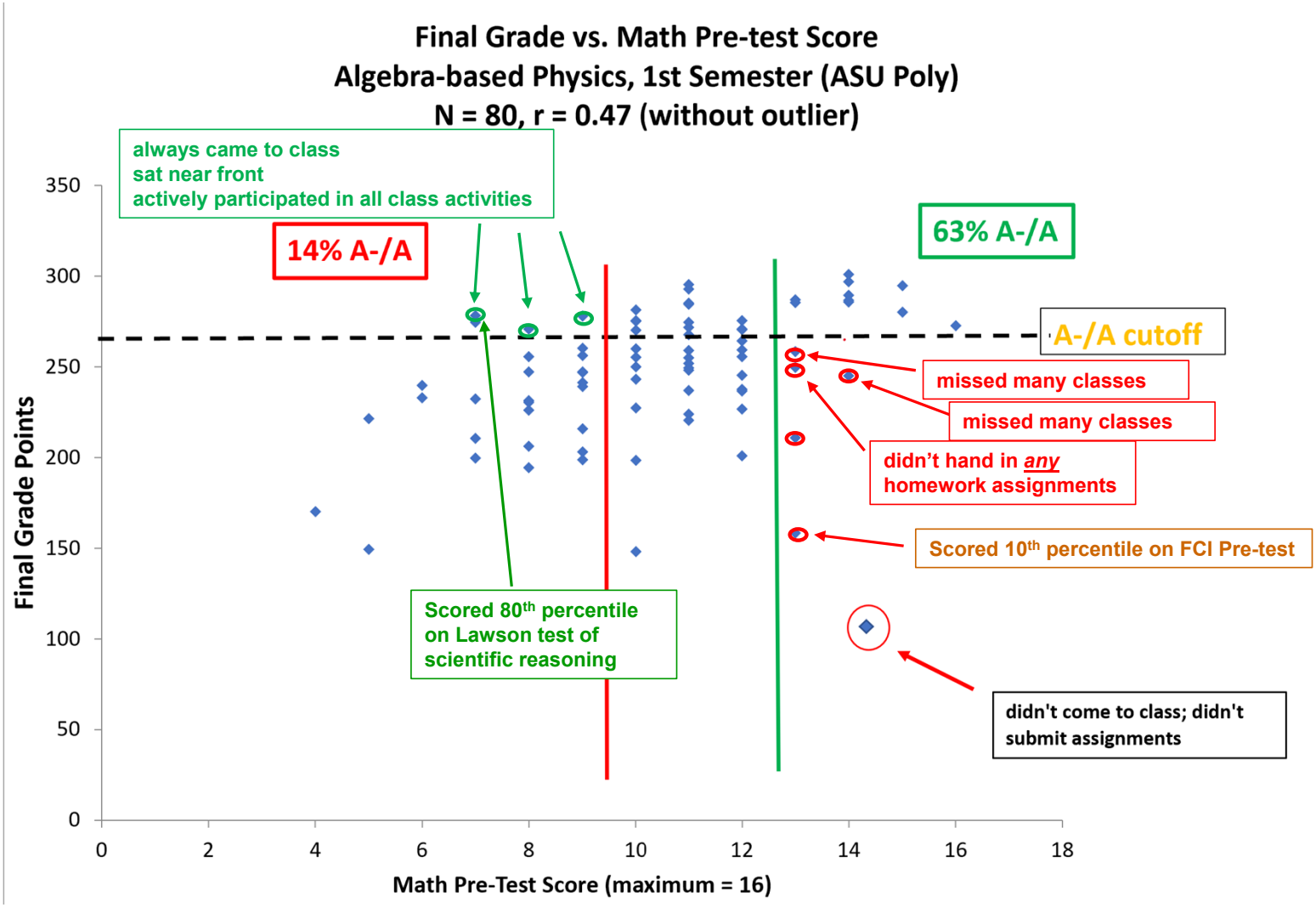

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

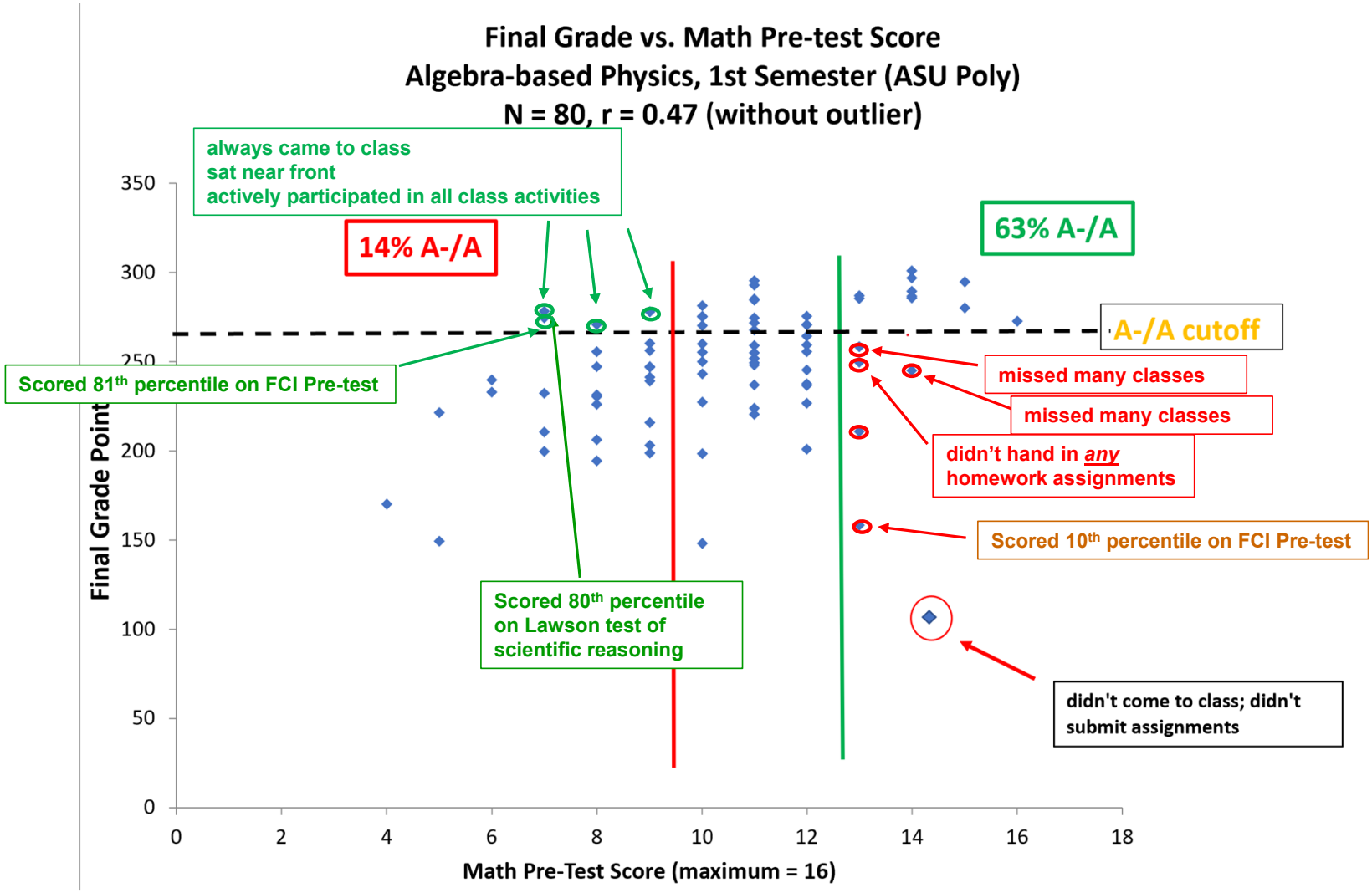

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

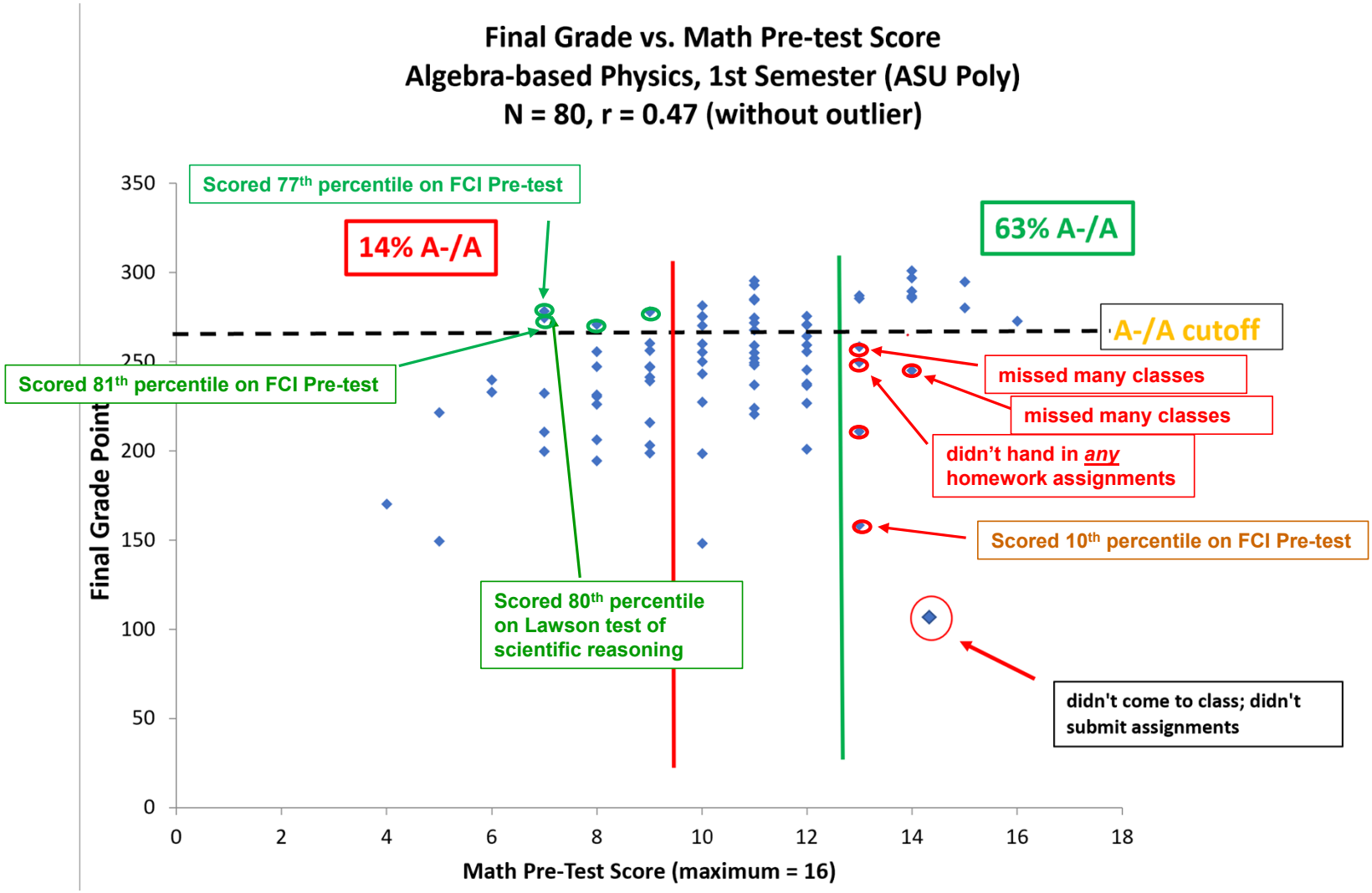

Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
Scored 17th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
Scored 17th percentile on Lawson test of scientific reasoning
didn't
homew
stopped coming to class
stopped taking quizzes
Scored 17th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
stopped coming to class
stopped taking quizzes
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
missed many classes
missed many classes
didn't hand in any homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Points
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
always came to class
sat near front
actively participated in all class activities
14% A-/A
63% A-/A
A-/A cutoff
Scored 81th percentile on FCI Pre-test
missed many classes
missed many classes
didn't hand in any homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Point
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18


Final Grade vs. Math Pre-test Score
Algebra-based Physics, 1st Semester (ASU Poly)
N = 80, r = 0.47 (without outlier)
Scored 77th percentile on FCI Pre-test
14% A-/A
63% A-/A
A-/A cutoff
Scored 81st percentile on FCI Pre-test
missed many classes
missed many classes
didn't hand in any homework assignments
Scored 10th percentile on FCI Pre-test
Scored 80th percentile on Lawson test of scientific reasoning
didn't come to class; didn't submit assignments
Final Grade Point
Math Pre-Test Score (maximum = 16)
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

Summary

- Performance on individual mathematics test items is predictive of overall diagnostic performance, and performance on the full diagnostic is somewhat predictive of final course grades
- Preliminary evidence suggests that “exceptions to the rule” regarding predictability of course performance may be explainable by motivational factors