Instructional implications of findings on students' mathematical difficulties

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Supported in part by NSF DUE #1504986 and #1914712

Acknowledgments

- Student data and very useful discussions have been provided by (among others):

  Darya Dolenko (Arizona State University)

  Andrew Heckler (Ohio State University)

  Beatriz Burrola Gabilondo (Ohio State University)

  Steven Pollock (University of Colorado, Boulder)

  Colin West (University of Colorado, Boulder)

  Christopher Varney (University of West Florida)

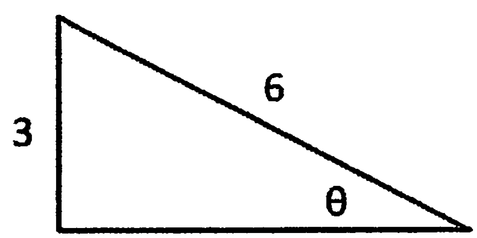
Overview

We have given diagnostic tests covering pre-college mathematics to over 7000 introductory physics students:

- Error rates were large enough to suggest that math difficulties can interfere with course performance;
- Results from five campuses at four different state universities were consistent with each other;
- We have adjusted our own instruction based on the findings, and offer some suggestions for other physics instructors.

Examples of Test Items

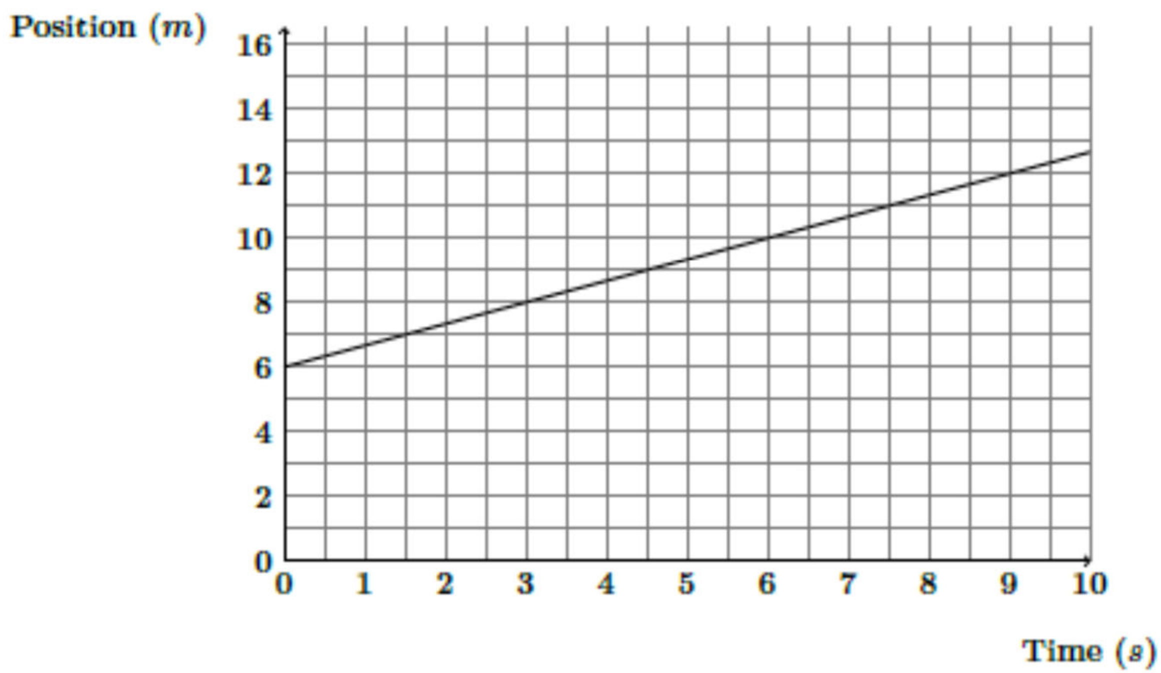
Find Unknown Angle



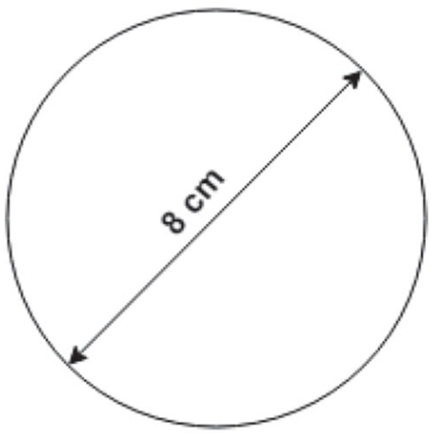
What is the value of θ?

Find Slope of Graph

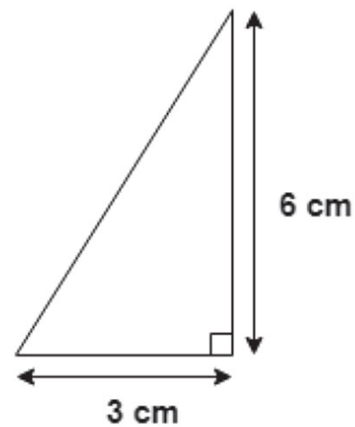
What is the slope of the graph below?



Find Area



(a) **Area of the circle =**



(b) **Area of the triangle =**

Simultaneous Equations, Symbolic Coefficients

$$cy = dx$$

$$a - y = bx$$

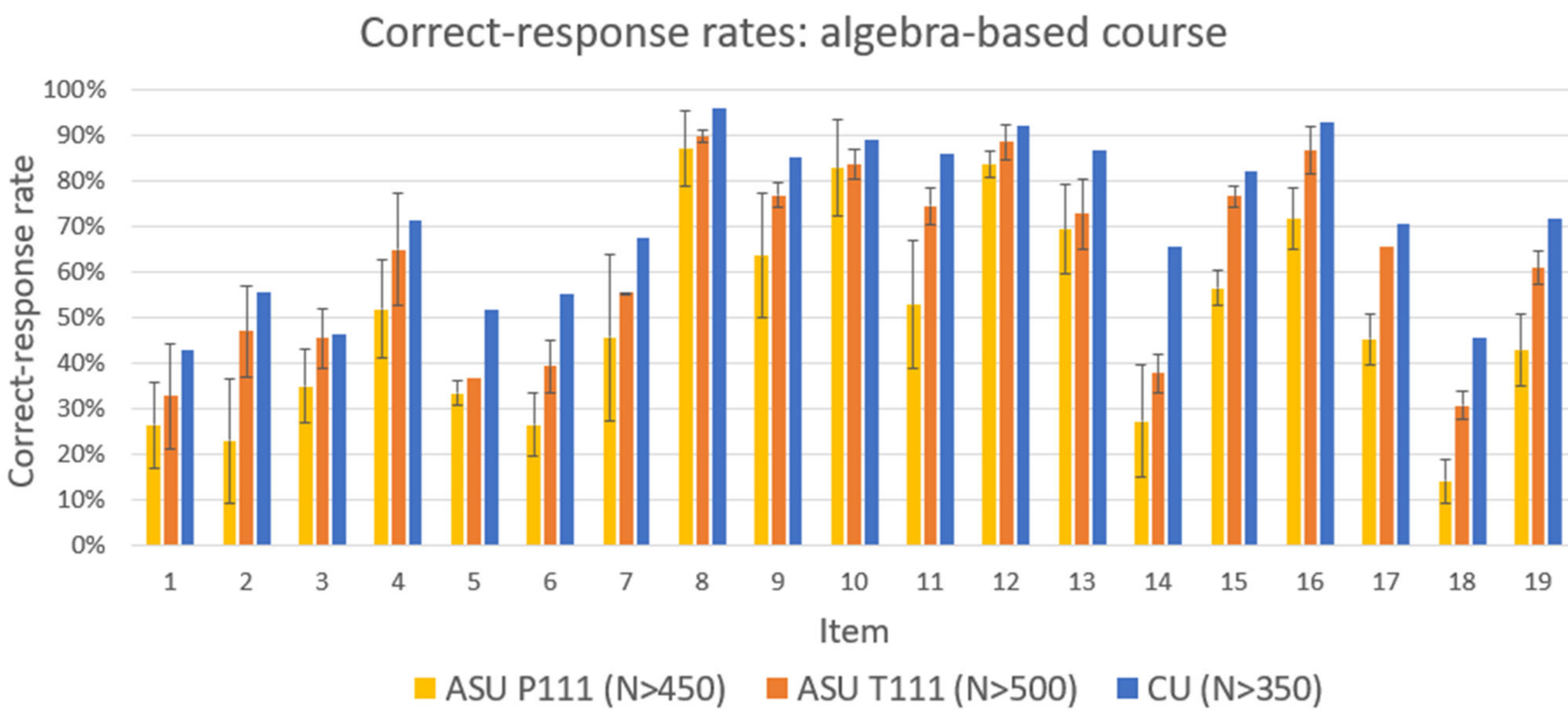
$$x = ?$$

1. High error rates on many items

- Error rates of 30-60% appear consistently among diverse test items in all student populations.

Implication: Instructors may need to adjust expectations of students' operational abilities with trigonometry, graphing, algebra, etc.

High consistency of results among five campuses at four different universities (three campuses shown below) suggests findings are generalizable



2. Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations significantly lowered students’ correct-response rate.

Correct-response rates are ≈ 25% lower on "symbolic" versions

Algebra: Simultaneous Equations (calculus-based course)

$0.5y = 2x$ $78.4 - y = 8x$	[Solve for x]	Numeric Version	79% correct (N = 1043)

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$cy = dx$ $a - y = bx$	[Solve for x]	Symbolic Version	55% correct (N = 862)

2. Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations significantly lowered students' correct-response rate.

Implication: Instructors may choose to be much more cautious in using symbolic manipulation to explain or demonstrate concepts.

3. Students favor non-standard solution methods

- Introductory physics students favor semi-arithmetic methods for solving solve algebraic equations; they do not "isolate the unknown variable."

Implication: Physics instructors' habitual approach to algebraic manipulation may be confusing to their introductory students.

13. What is the numerical value of d?

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

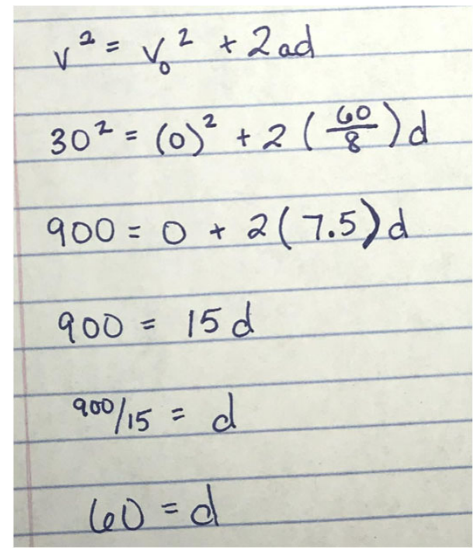
$$v = 30$$

$d = ?$ How would you solve this?

(Please clearly *circle* your answer and show all work.)

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

53/53 students solved it this way:



4. Similar error rates on different topics

- Students' errors on specific topics were highly correlated with errors on other, disparate topics (e.g., trigonometry, geometry, graphing, algebra).

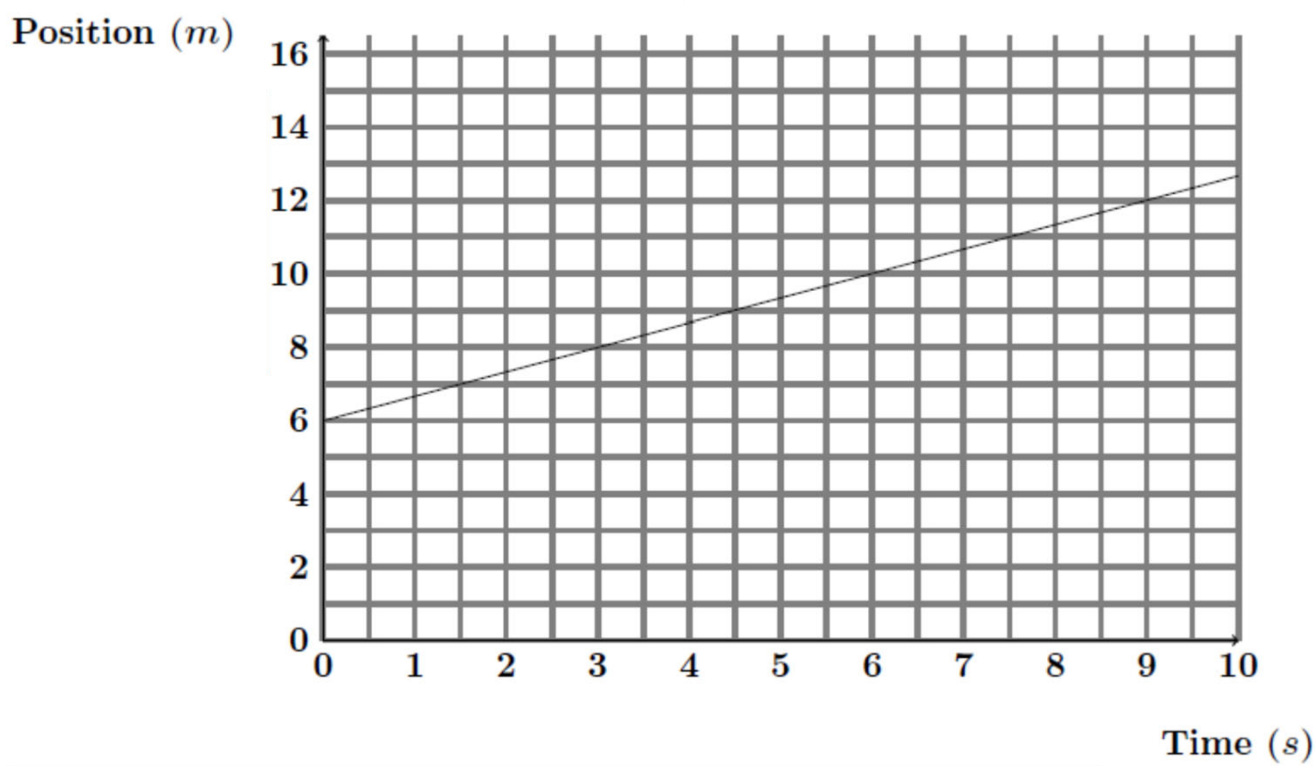
Implication: If instructors are aware that their students have difficulties with a specific type of mathematical operation, they may be confident that the students will have analogous difficulties with *other* types of operations.

5. Students show weakness with units and graphing

- Many students in both algebra- and calculus-based physics courses are extremely weak in handling units and/or graphs: they ignored graph-axis labels, and provided no or incorrect units for area and velocity.

Implication: Instructors may not fully appreciate the degree to which many students are challenged in using units and graphs.

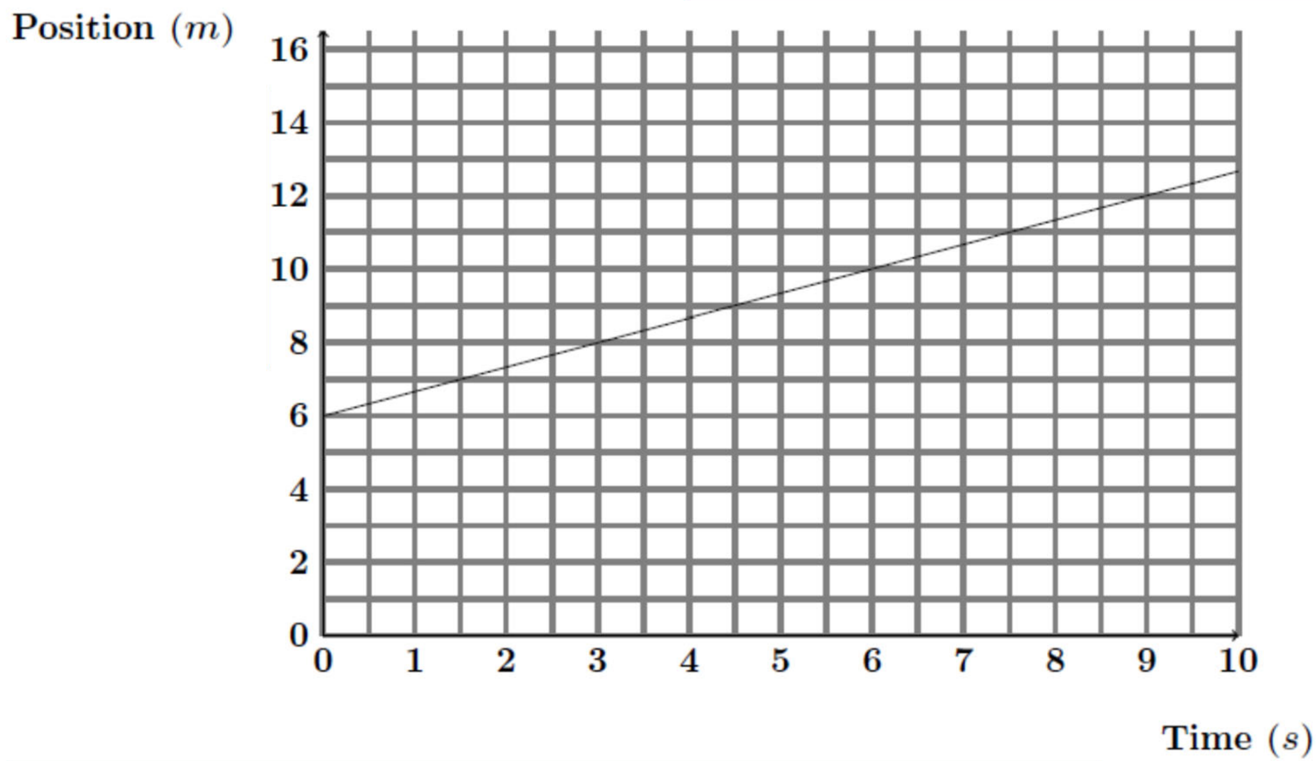
What is the slope of the graph below?



What is the slope of the graph below?

Correct-response rate (N > 2000):

30-60%, nearly independent of course or campus



What is the slope of the graph below?

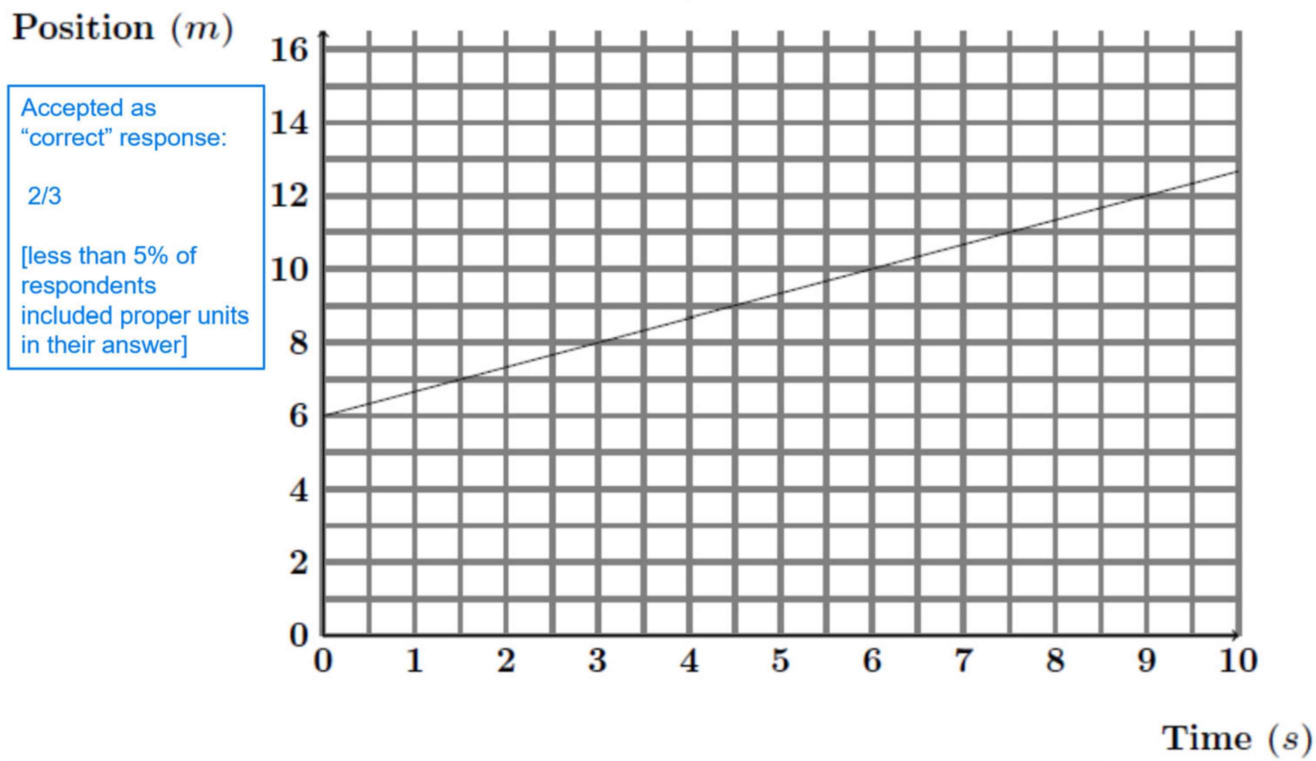
Correct-response rate (N > 2000):

30-60%, nearly independent of course or campus

Accepted as "correct" response:

2/3

[less than 5% of respondents included proper units in their answer]



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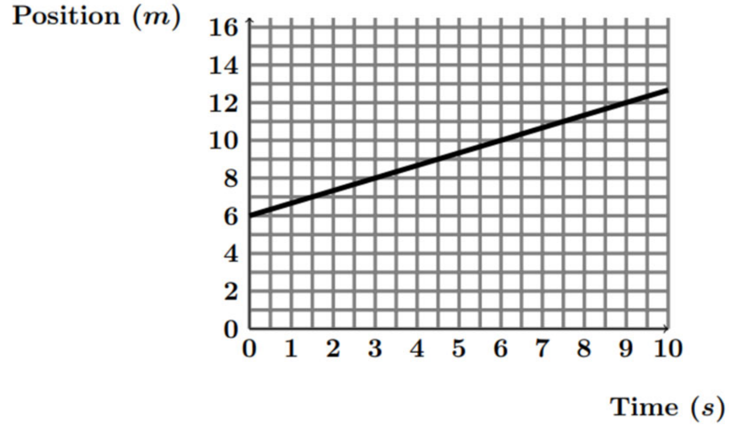


A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

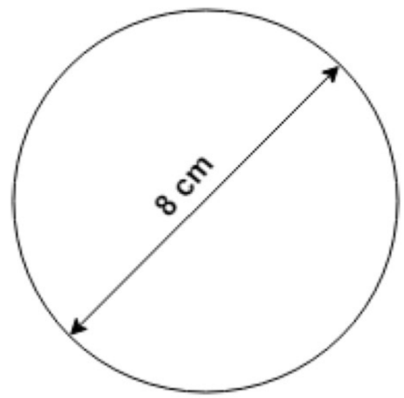
***N* = 2556**

Numerically correct (C or D): 59%

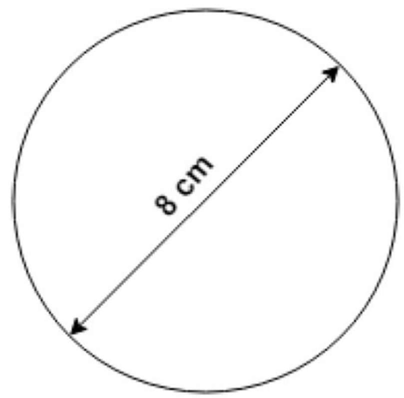
***Actually* correct (C): 48%**

Consistent with results on written version

Most common error: *Counting grid squares and ignoring numbers on axes*

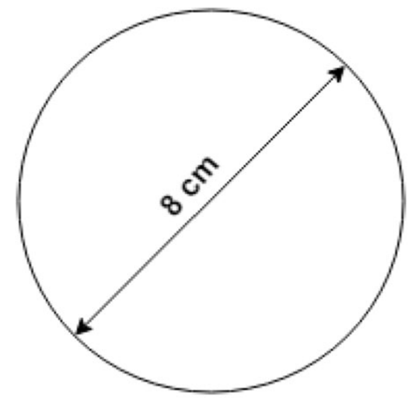


(a) Area of the circle =



(a) Area of the circle =

Area of Circle: Algebra- and Calculus-based courses combined, 2018



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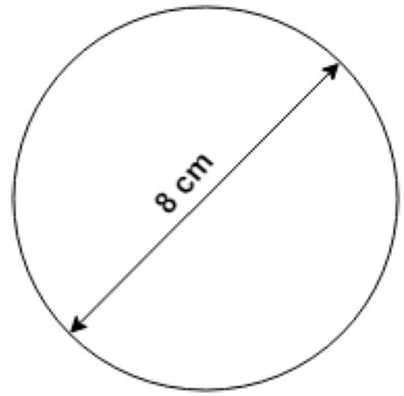
Area of Circle: Algebra- and Calculus-based courses combined, 2018

ASU-Poly: 57% correct (N = 250)
ASU-Tempe: 76% correct (N = 1086)



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Area of Circle: Algebra- and Calculus-based courses combined, 2018

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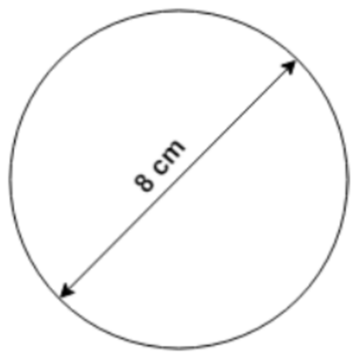
ASU-Tempe: 76% correct (N = 1086)

…*with correct units:* 29% and 45% correct, respectively

On-line Version:



(a) Area of the circle = ?

A. 8π cm	F. 8π cm^2	K. 8π cm^3
B. 16π cm	G. 16π cm^2	L. 16π cm^3
C. 32π cm	H. 32π cm^2	M. 32π cm^3
D. 64π cm	I. 64π cm^2	N. 64π cm^3
E. 128π cm	J. 128π cm^2	O. 128π cm^3

20% did *not* choose cm^2

(N = 1252)

6. Students make many “careless” errors

- During interviews, students tended to self-correct approximately 60% of their initial errors, suggesting many errors are “careless.”

Implication: Instruction on error-detecting, checking, and self-correcting strategies may offer disproportionately high returns in helping students address their mathematical difficulties.

7. Even single test items are highly predictive

- Class-average scores on even a *single* diagnostic test item—regardless of which item was chosen—were highly predictive of average scores on 13 other diagnostic items covering varied topics.

Implication: It may be possible to diagnose the level of students' difficulties with only one or very few mathematics pretest items.

Predictability at Whole-Class Level

- Performance on **one single diagnostic item** can *accurately* predict class-average score on full 13-item diagnostic

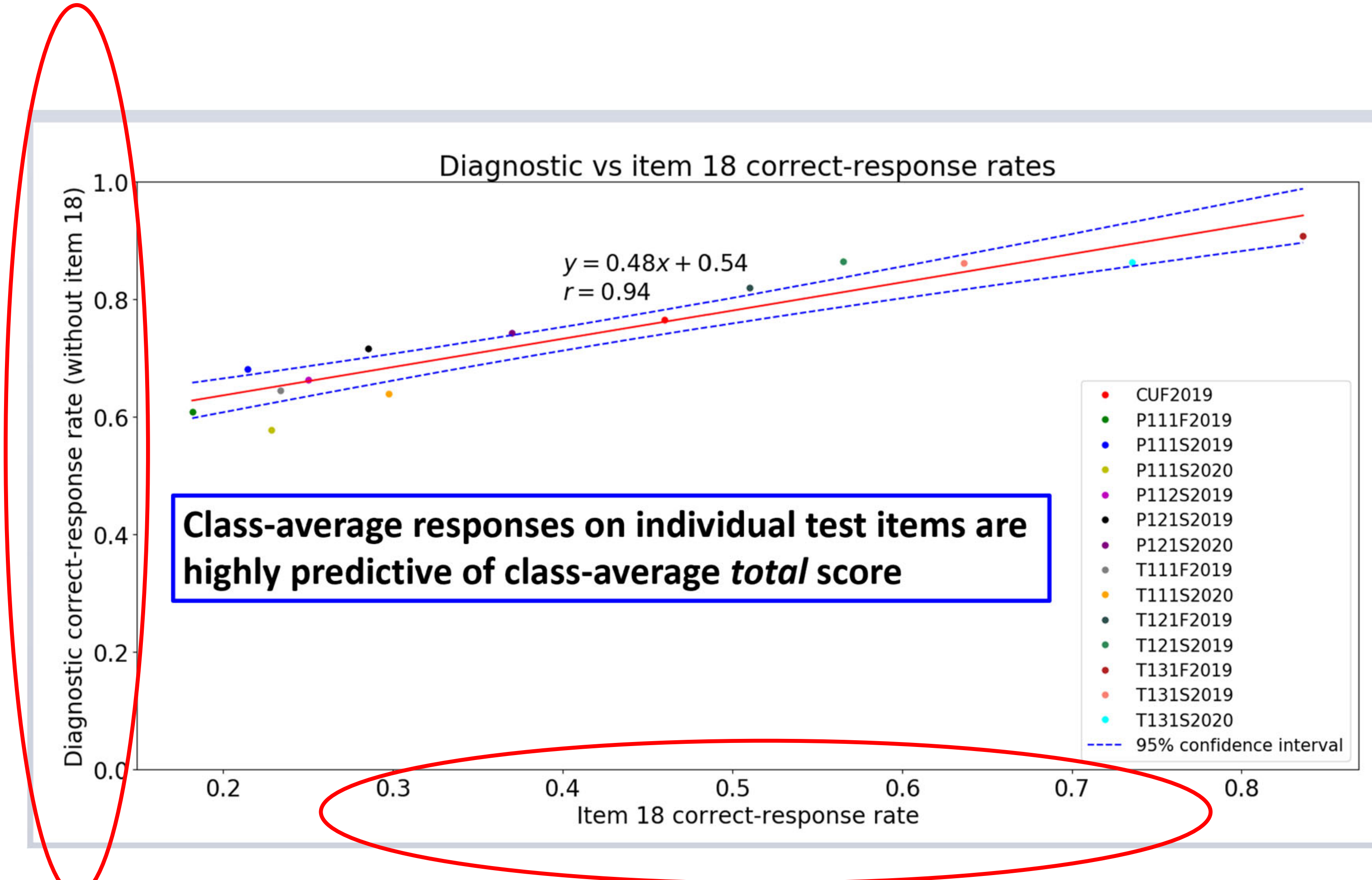
Example:

[#18]

18. $cy = dx$

$$a - y = bx$$

$$x = ?$$


Diagnostic vs item 18 correct-response rates
y = 0.48x + 0.54
r = 0.94
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval
Class-average responses on individual test items are highly predictive of class-average total score

8. Math performance somewhat predictive of final grade

- Limited data: two class samples
- Clear pattern, but pattern type depends on student population
- No evidence of *causal* relationship

Predictability at Individual-Student Level

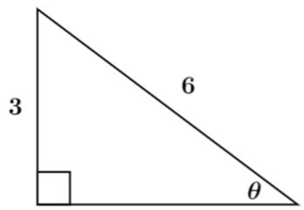
- Performance on **3-item subset** can *approximately* predict final course grade

Example:

[#3, #11, #12]

What is the value of θ?

#3



A. $cos(3/6)$ D. $cos^{-1}(3/6)$ G. $30°$ J. $27°$

B. $sin(3/6)$ E. $sin^{-1}(3/6)$ H. $45°$ K. $3/6$

C. $tan(3/6)$ F. $tan^{-1}(3/6)$ I. $60°$ L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

#12

$$\frac{3}{2} = 7x$$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$\frac{a/b}{c^2/d} = ?$$

#11

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

Calculus-based Physics, 1st semester (UWF)

N = 95, 32% with final grade B+/A-/A

0 or 1 correct on [#3, #11, #12]	3/3 correct on [#3, #11, #12]
(*N* = 21)	(*N* = 44)
5% with final grade B+/A-/A	52% with final grade B+/A-/A

Predictability at Individual-Student Level

- Performance on **full online diagnostic** can *approximately* predict final course grade

Examples:

Calculus-based physics, 1st semester (UWF)

Algebra-based physics, 2nd semester (ASU Tempe)

Calculus-based Physics, 1st semester (UWF)

N = 101, 30% with final grade B+/A-/A

<70% correct responses (full diagnostic) (*N* = 35)	>92% correct responses (full diagnostic) (*N* = 21)
6% with final grade B+/A-/A	62% with final grade B+/A-/A

Algebra-based Physics, 2nd semester (ASU Tempe)

N = 118, 59% with final grade A-/A/A+

<86% correct responses (full diagnostic) (*N* = 101)	>92% correct responses (full diagnostic) (*N* = 17)
53% with final grade A-/A/A+	94% with final grade A-/A/A+

Summary

- The scale of physics students' difficulties with basic mathematical operations may warrant adjustment of instructors' expectations and instructional approach
- Performance on individual mathematics test items is predictive of overall diagnostic performance, and somewhat predictive of final course grades