

Symbolic manipulation fluency
predicts introductory physics
students' mathematical
preparedness

Dakota H. King and David E. Meltzer

Arizona State University

Supported in part by NSF DUE #1504986 and #1914712

Our mathematics diagnostic

- Administered to 7,264 students over the past 5 years
 - Includes students enrolled in algebra- and calculus-based introductory physics courses at four large state universities
- Tests knowledge on basic trigonometry, geometry, graphing, and algebra
- Our latest version:
 - A multiple-choice online assessment
 - In addition to math, it includes physics items testing conceptual understanding of Newton's second and third laws



Our mathematics diagnostic

Link to our diagnostic: <https://testmoz.com/q/ExampleDiagnostic>

Previous **hand-written** versions (in total):

- 6061 students
- 38 classes
- 10-15 mathematics items
- Free-response and multiple-choice items



Latest **online** version:

- 1203 students
- 11 classes
- 14 mathematics items
- 4 physics items
- Only multiple-choice items

Designing the online diagnostic

- Free-response items were reformatted to multiple-choice by analyzing student responses from four years of data

Hand-written
version

$$cy = dx$$
$$a - y = bx$$

$$x = ?$$



Online version

$$cy = dx$$
$$a - y = bx$$

$$x = ?$$

A.	$\frac{ac}{d+b}$	C.	$\frac{ac}{bc-d}$	E.	$\frac{ac}{db}$	G.	$\frac{a}{b+\frac{d}{c}}$	I.	$\frac{1}{b}\left(a-\frac{d}{c}\right)$
B.	$\frac{ac}{d-b}$	D.	$\frac{ac}{bc+d}$	F.	$\frac{a}{db}$	H.	$\frac{a}{b+d}$	J.	$\frac{c}{d}(a-b)$

Designing the online diagnostic

Hand-written
version

Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

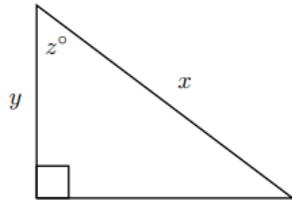
Online version

Solve for θ .

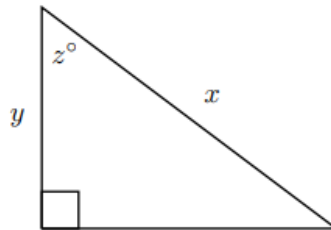
$$\gamma\theta + \eta = \lambda\theta + \omega$$

- A. $\frac{\eta + \omega}{\gamma - \lambda}$ C. $\frac{\gamma - \lambda}{\omega - \eta}$ E. $\frac{\eta - \omega}{\gamma\lambda}$ G. $\frac{\omega - \eta}{\gamma - \lambda}$ I. $\frac{\eta - \omega + \gamma}{\lambda}$
B. $\frac{\eta - \omega}{\lambda - \gamma}$ D. $\frac{\lambda - \gamma}{\eta - \omega}$ F. $\frac{\omega - \eta}{\gamma\lambda}$ H. $\frac{\omega - \eta}{\gamma + \lambda}$ J. $\frac{\omega - \eta + \lambda}{\gamma}$

What is the length of side x ?



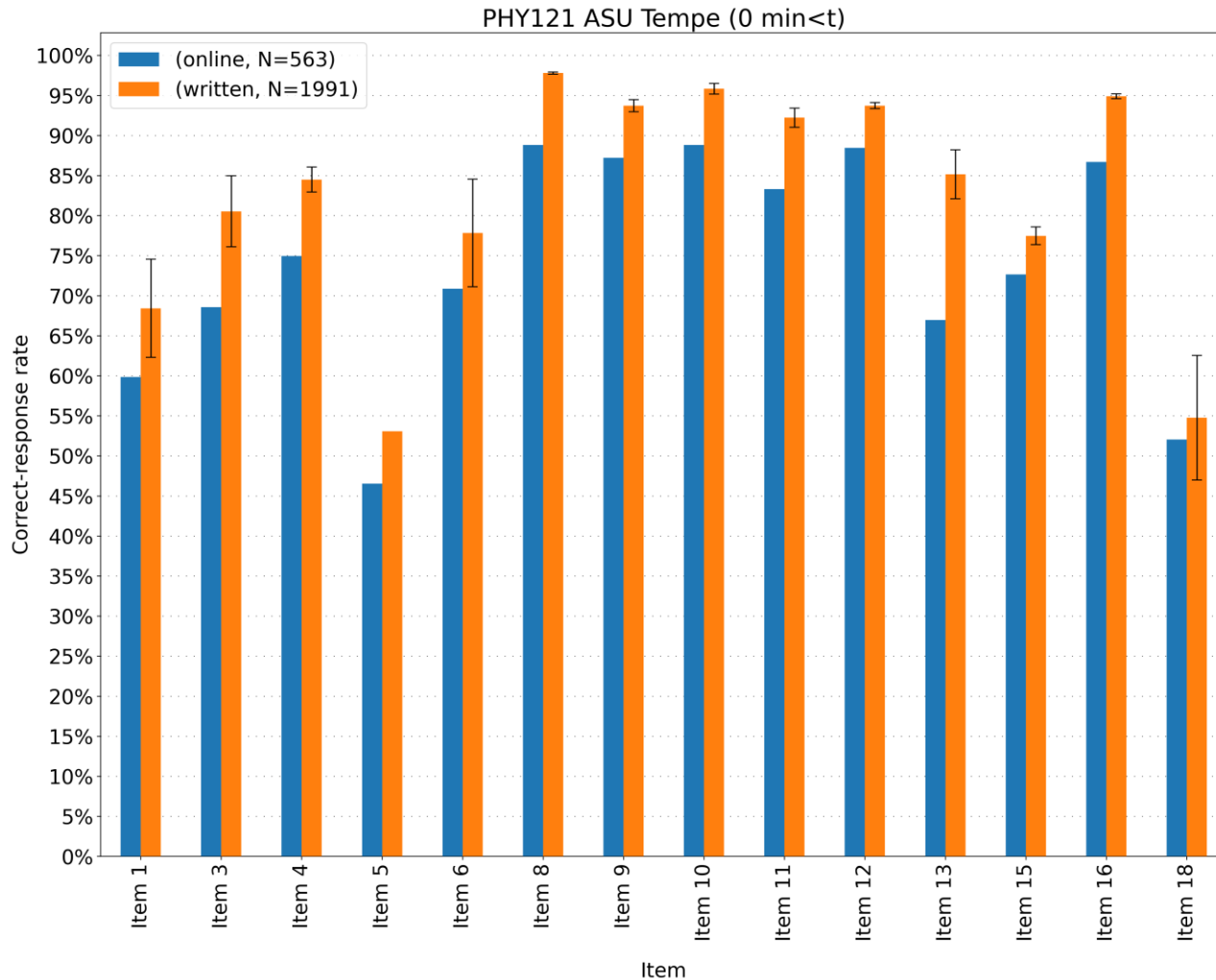
What is the length of side x ?



- A. $y\cos(z^\circ)$ D. $\frac{y}{\cos(z^\circ)}$ G. $\frac{\cos(z^\circ)}{y}$ J. $\sqrt{y^2 + z^2}$
B. $y\sin(z^\circ)$ E. $\frac{y}{\sin(z^\circ)}$ H. $\frac{\sin(z^\circ)}{y}$ K. $\sqrt{z^2 - y^2}$
C. $y\tan(z^\circ)$ F. $\frac{y}{\tan(z^\circ)}$ I. $\frac{\tan(z^\circ)}{y}$ L. y/z

Results: online vs hand-written diagnostic
(math items only)

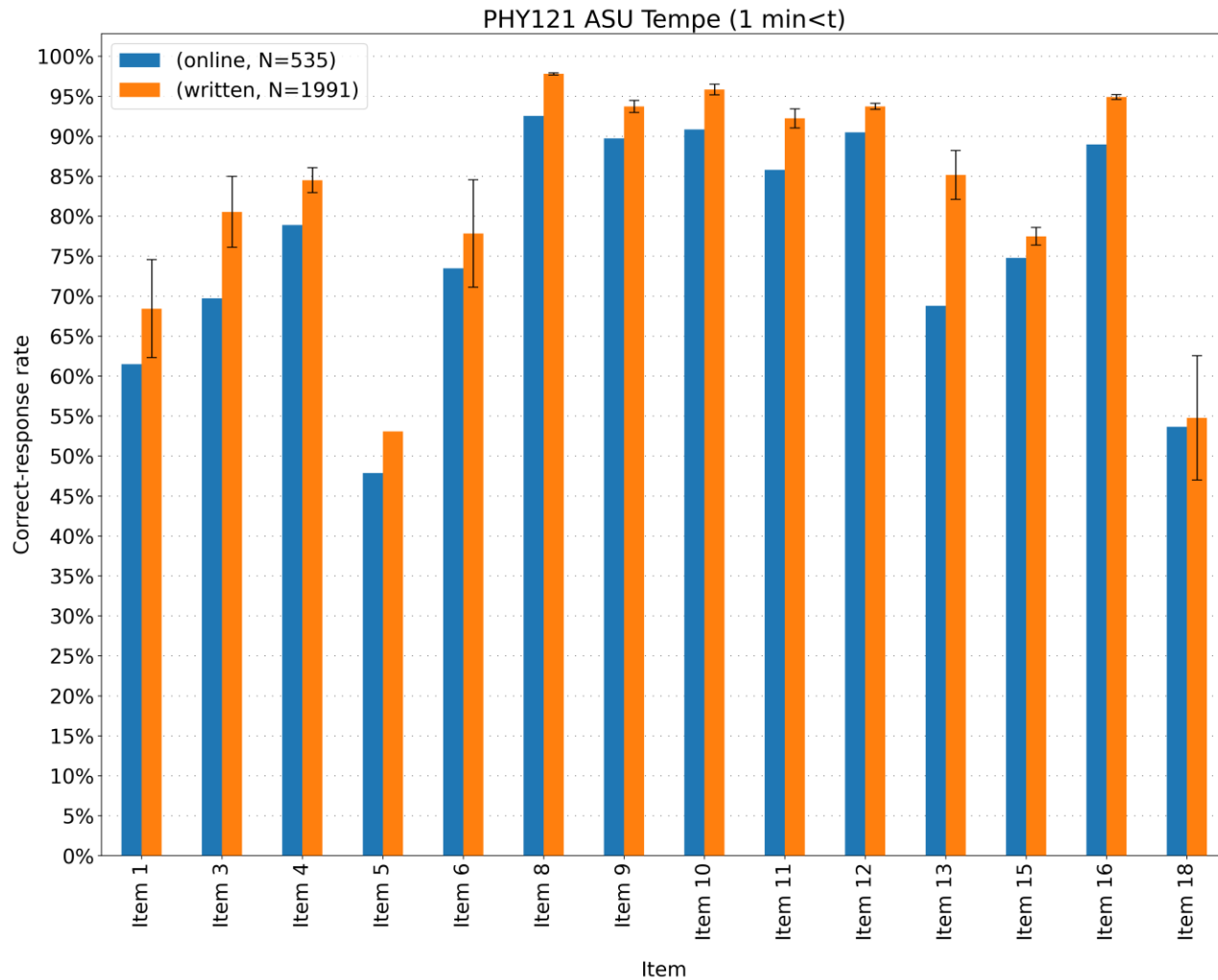
Online vs hand-written: correct-response rates



➤ Data before applying a time cutoff

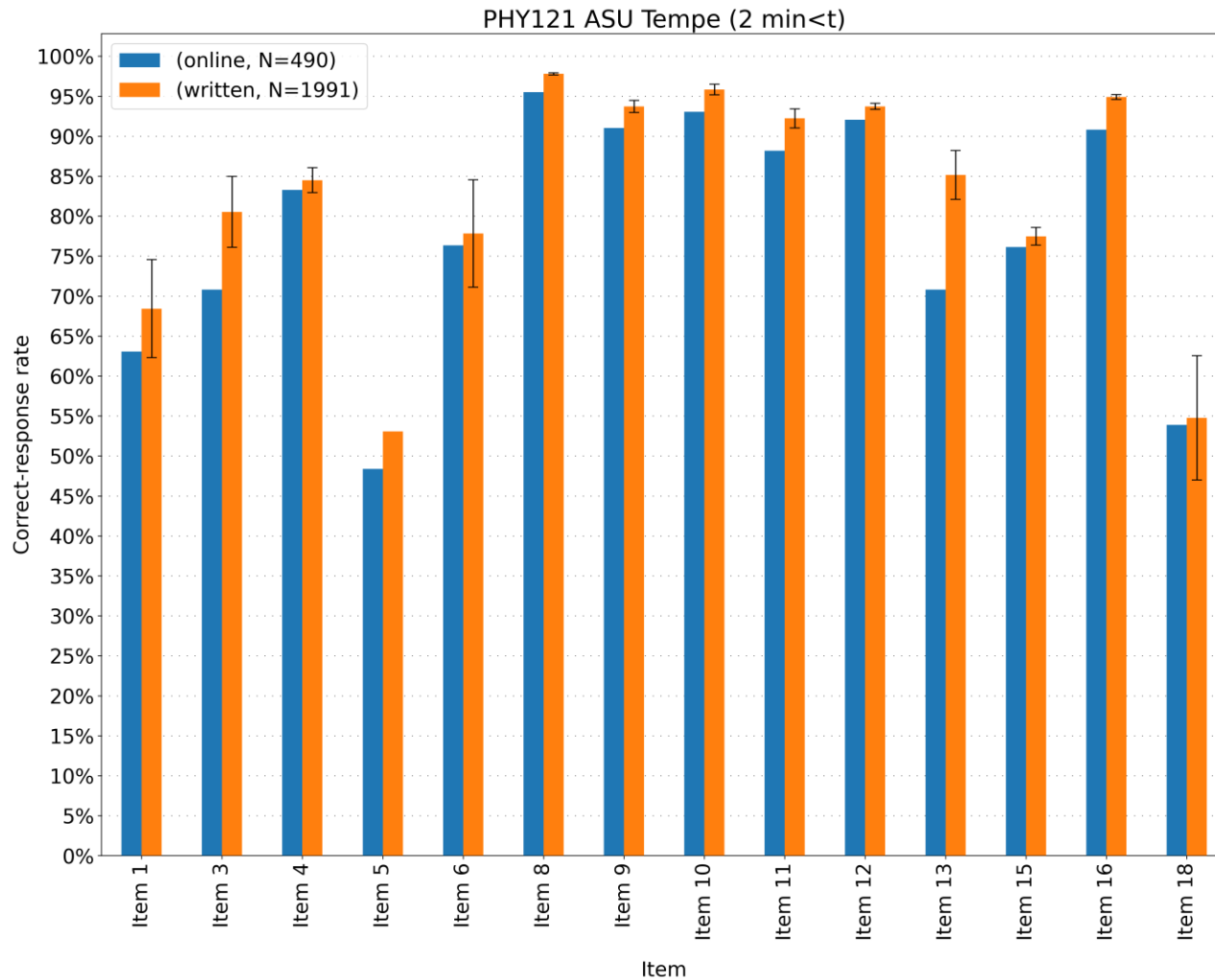
0 min < t

Online vs hand-written: correct-response rates



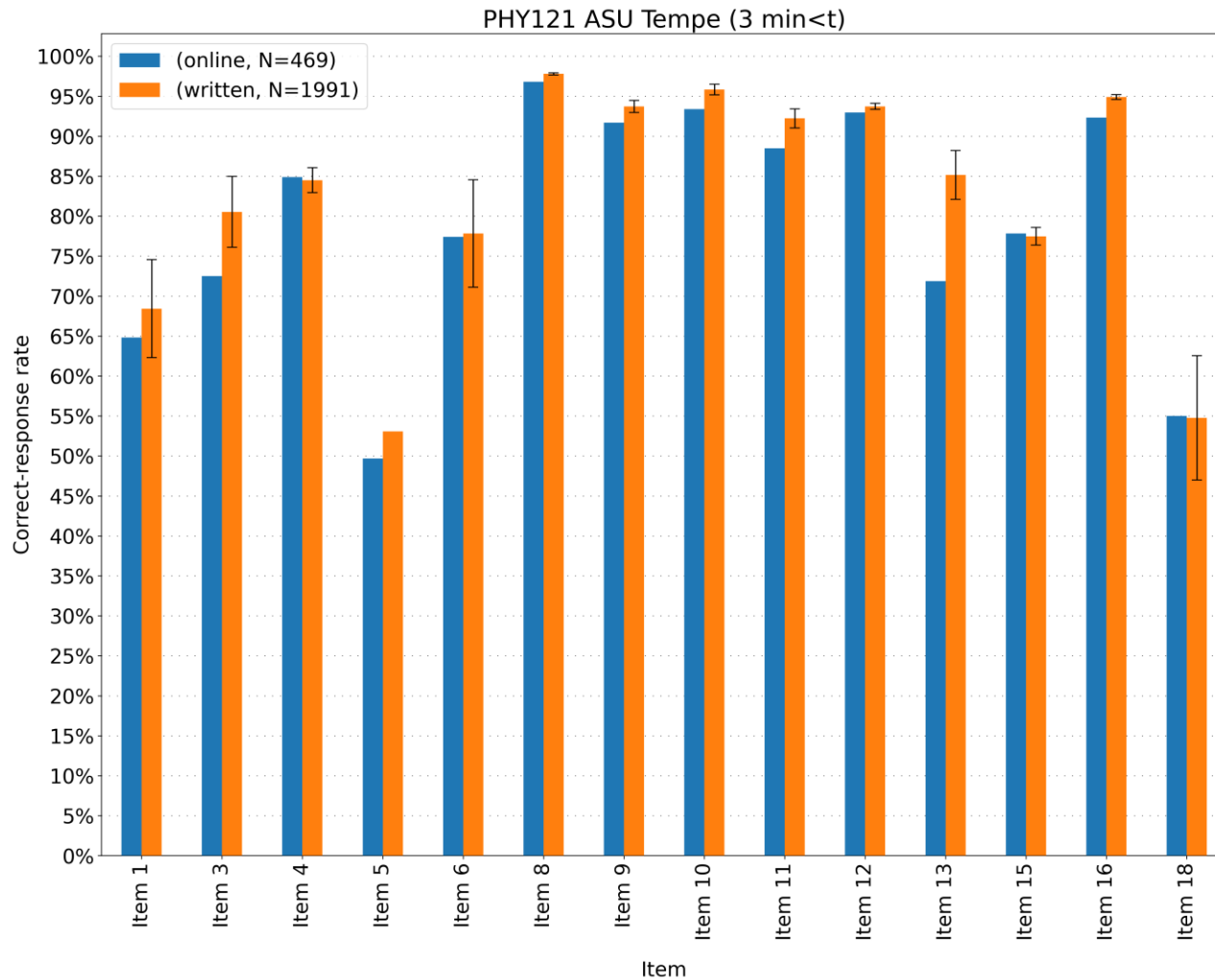
1 min < t

Online vs hand-written: correct-response rates



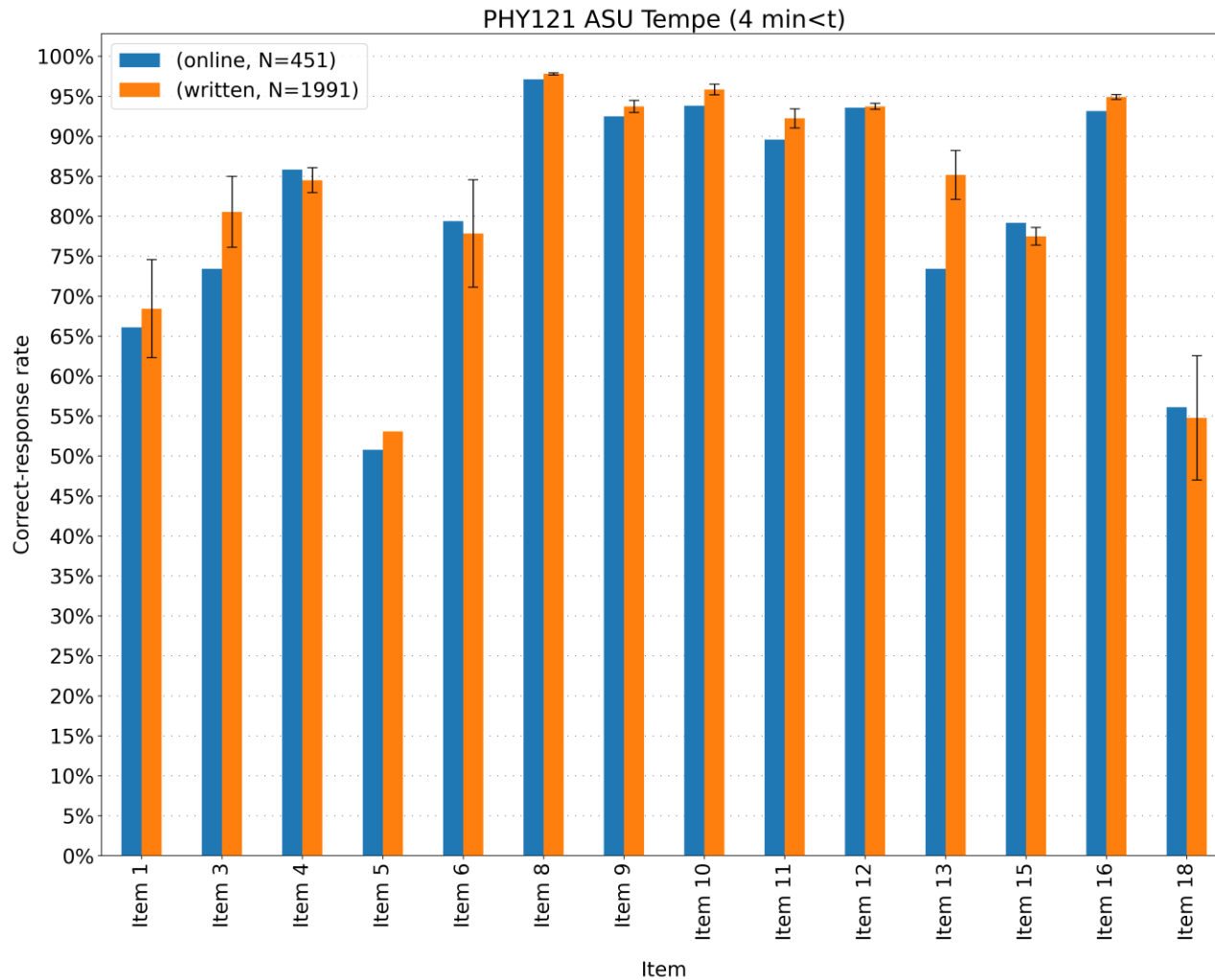
2 min < t

Online vs hand-written: correct-response rates



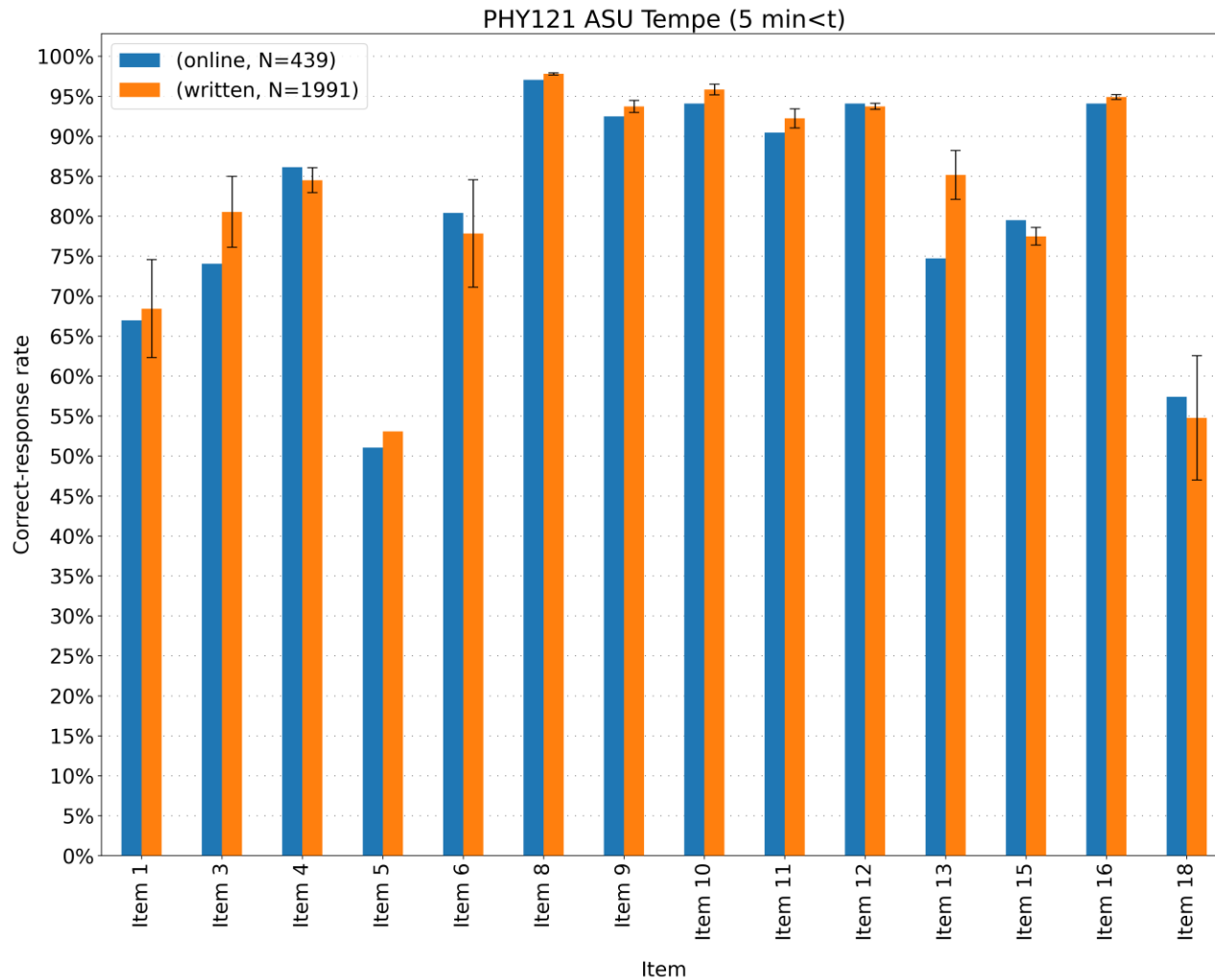
3 min < t

Online vs hand-written: correct-response rates



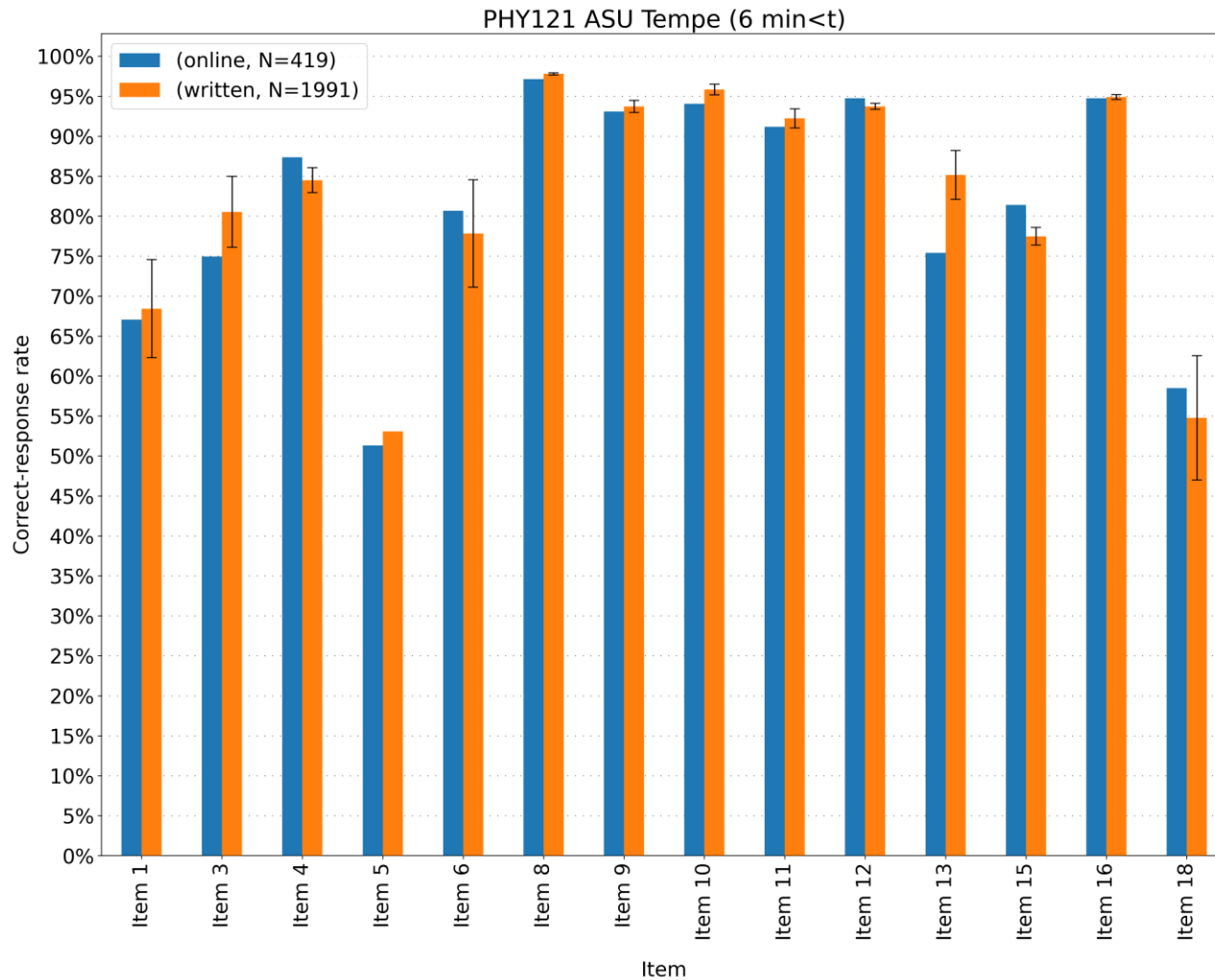
4 min < t

Online vs hand-written correct-response rates



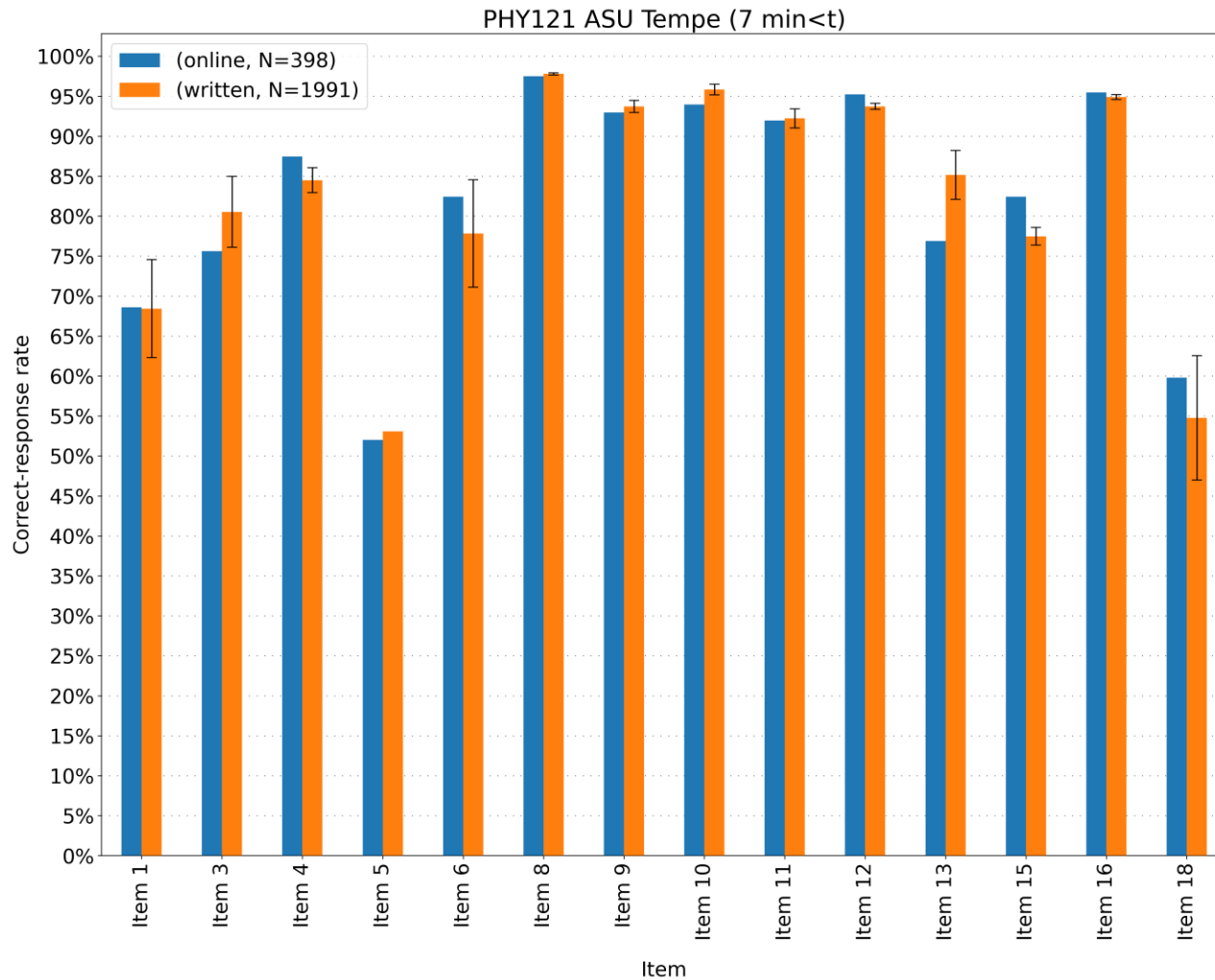
5 min < t

Online vs hand-written correct-response rates



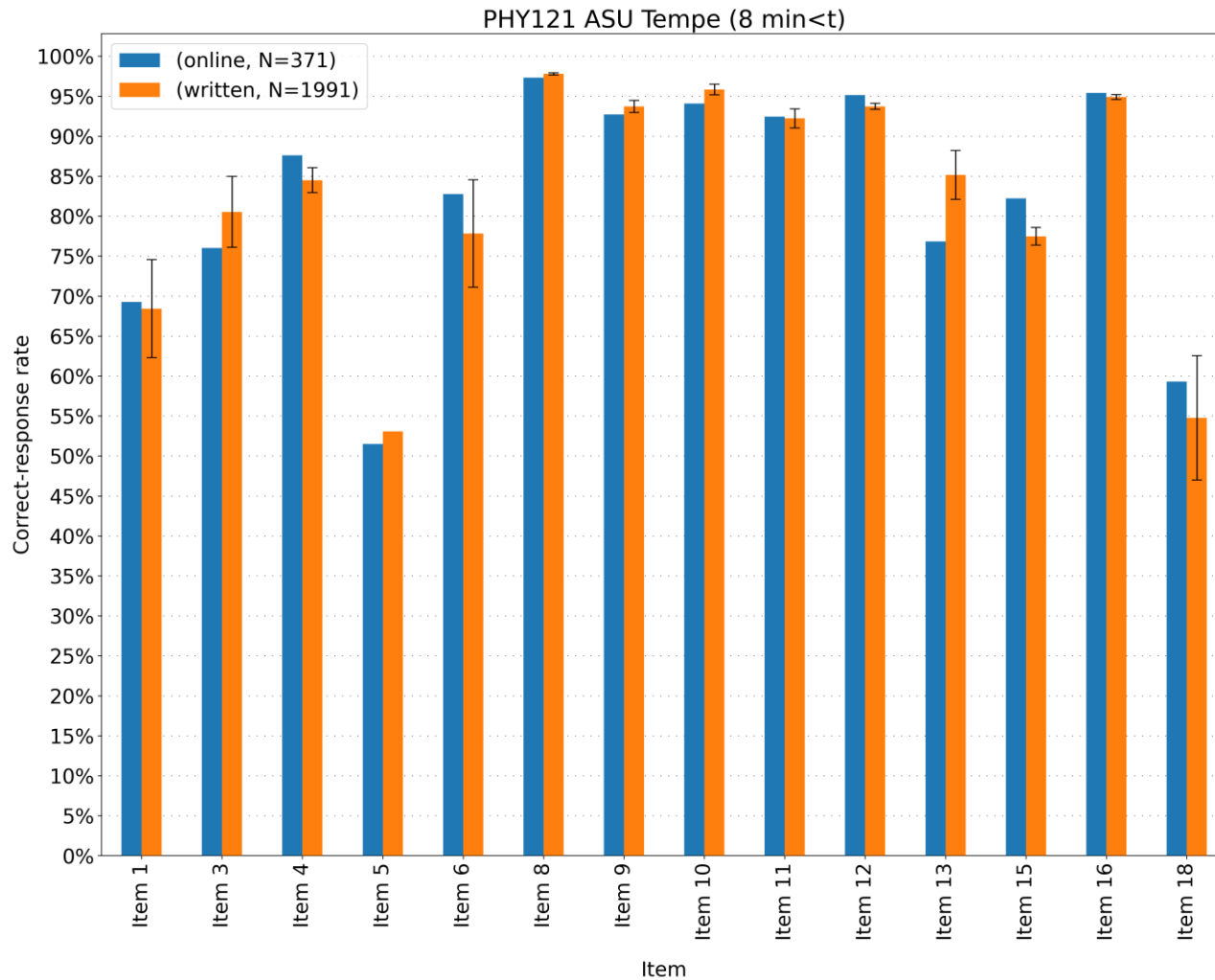
6 min < t

Online vs hand-written correct-response rates



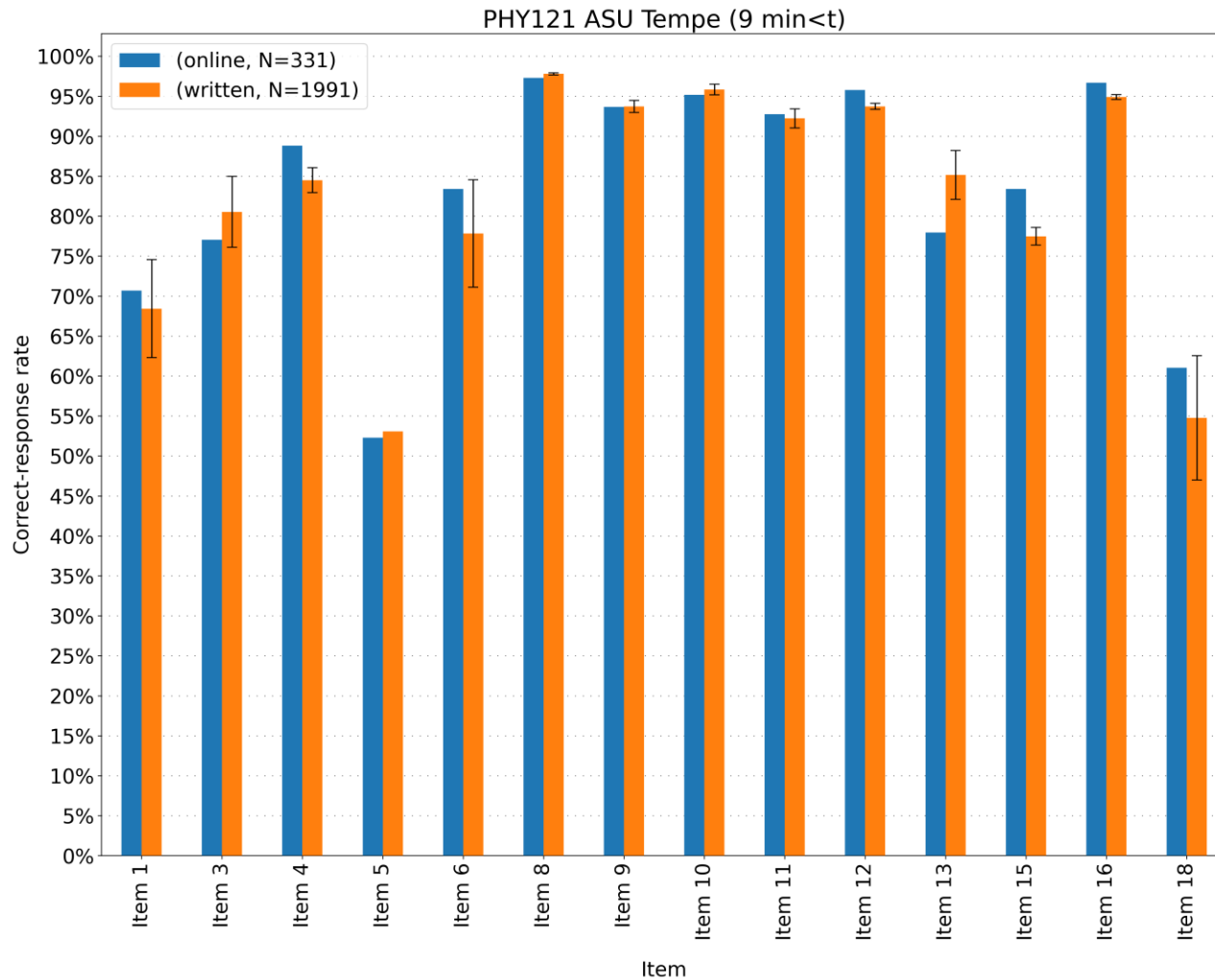
7 min < t

Online vs hand-written correct-response rates



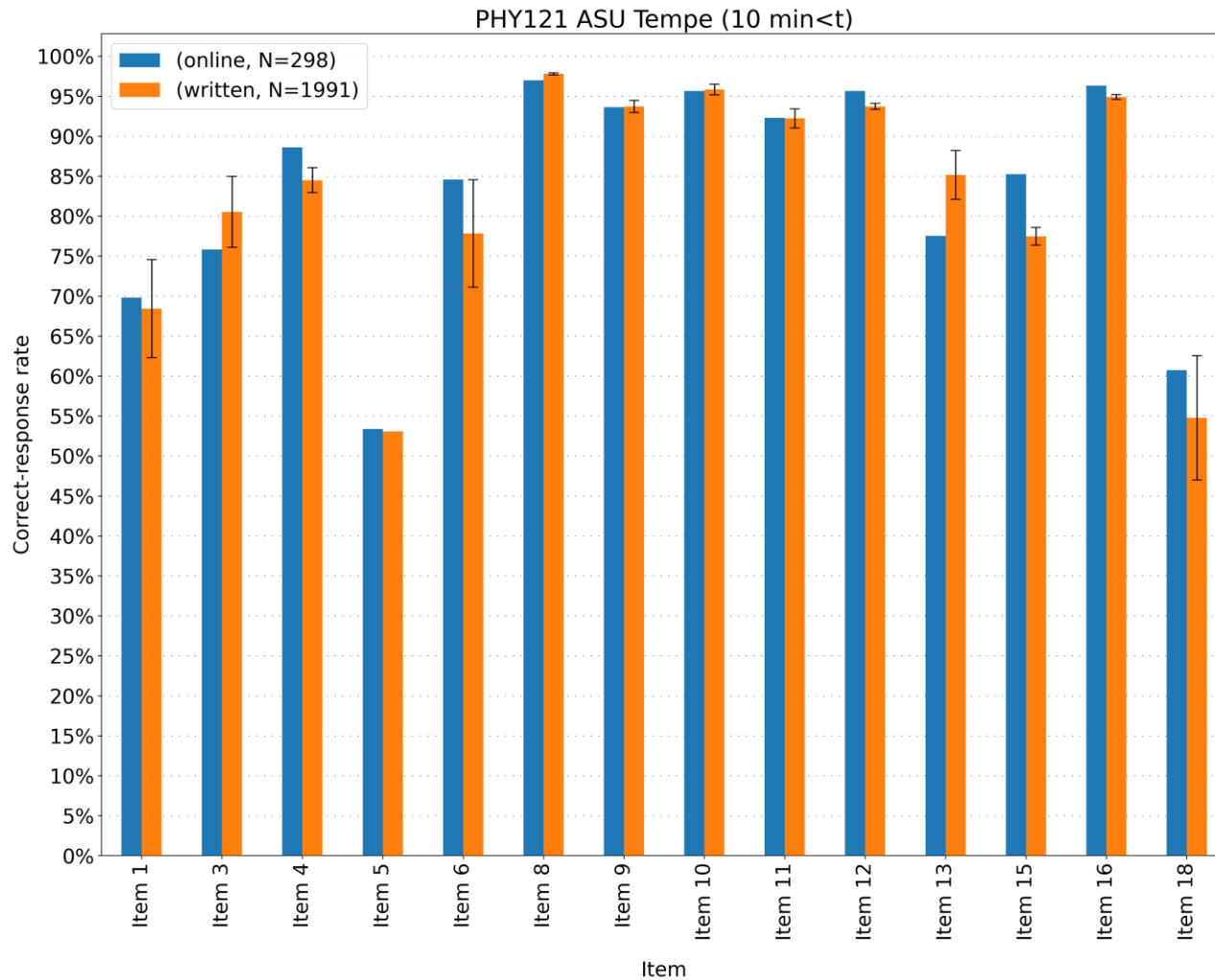
8 min < t

Online vs hand-written correct-response rates



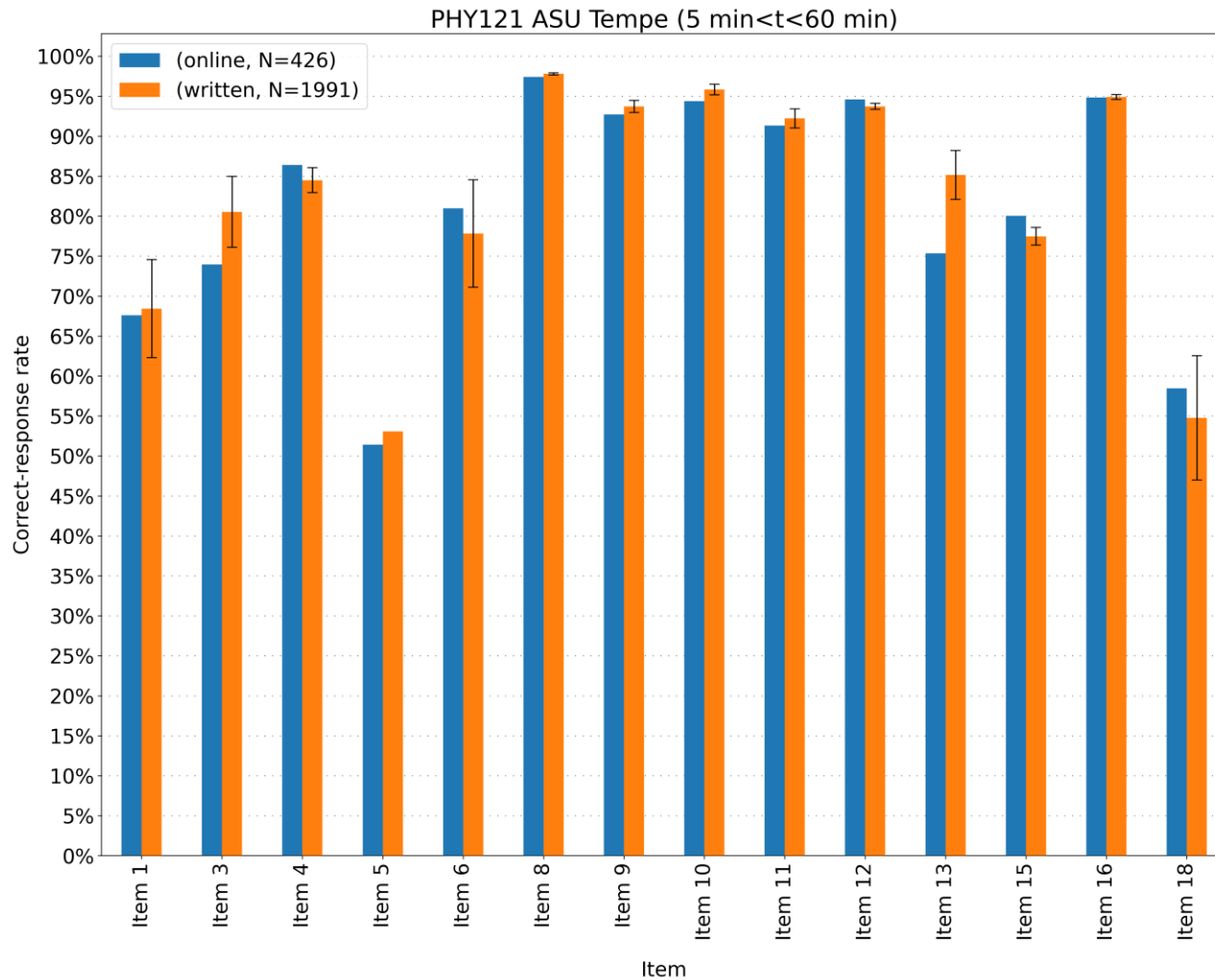
9 min < t

Online vs hand-written correct-response rates



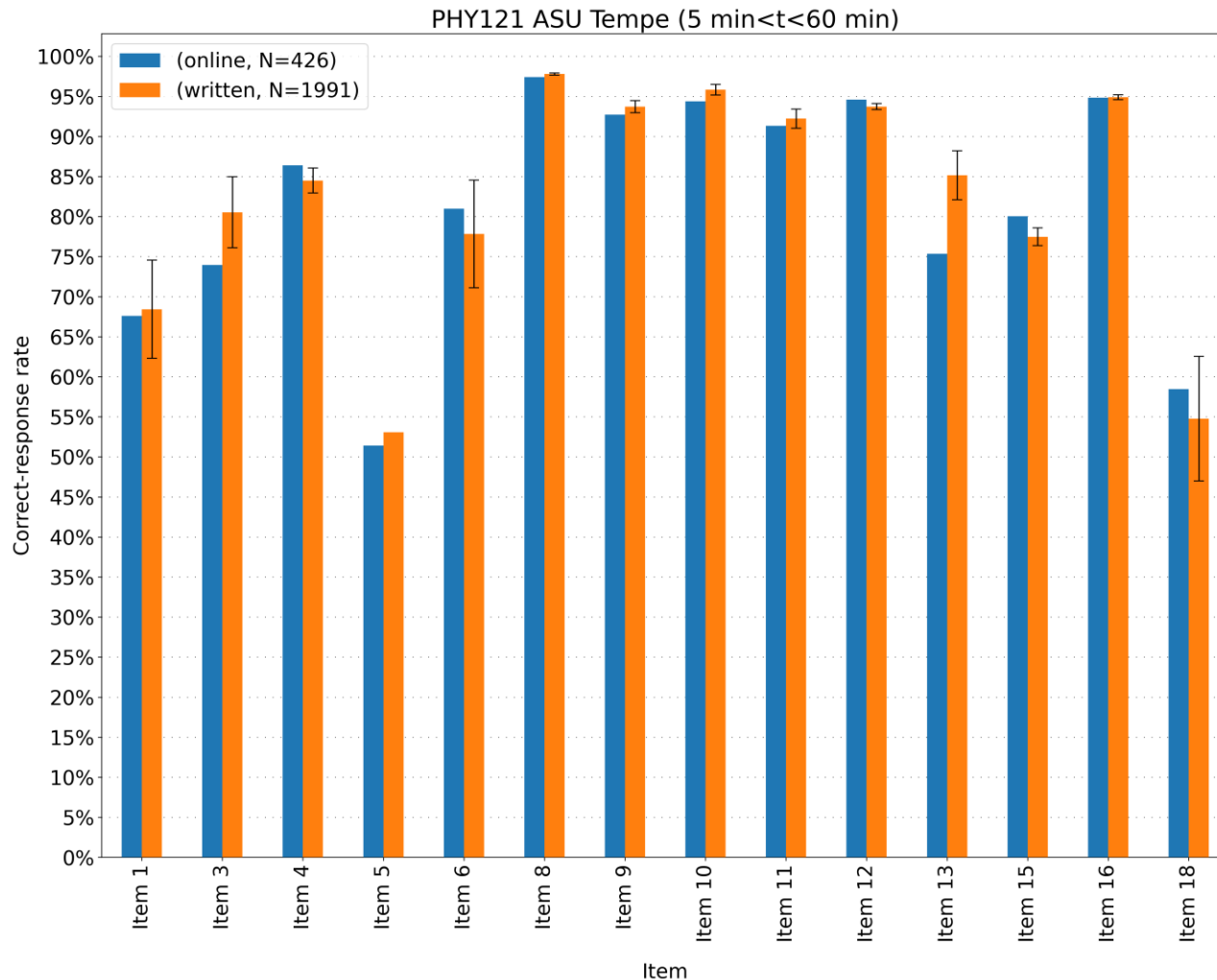
10 min < t

Online vs hand-written: correct-response rates



5 min < t < 60 min

Online vs hand-written: correct-response rates



- Applying a time cutoff helped reduce guessing rates
- Correct-response rates are within 6% for all items except item 13

Online vs hand-written: course-level predictive power

- With a class's score on a single item, can we predict the class's mean performance on the remaining 13 math items?

Item 18

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

A. $\frac{ac}{d+b}$

C. $\frac{ac}{bc-d}$

E. $\frac{ac}{db}$

G. $\frac{a}{b + \frac{d}{c}}$

I. $\frac{1}{b} \left(a - \frac{d}{c} \right)$

B. $\frac{ac}{d-b}$

D. $\frac{ac}{bc+d}$

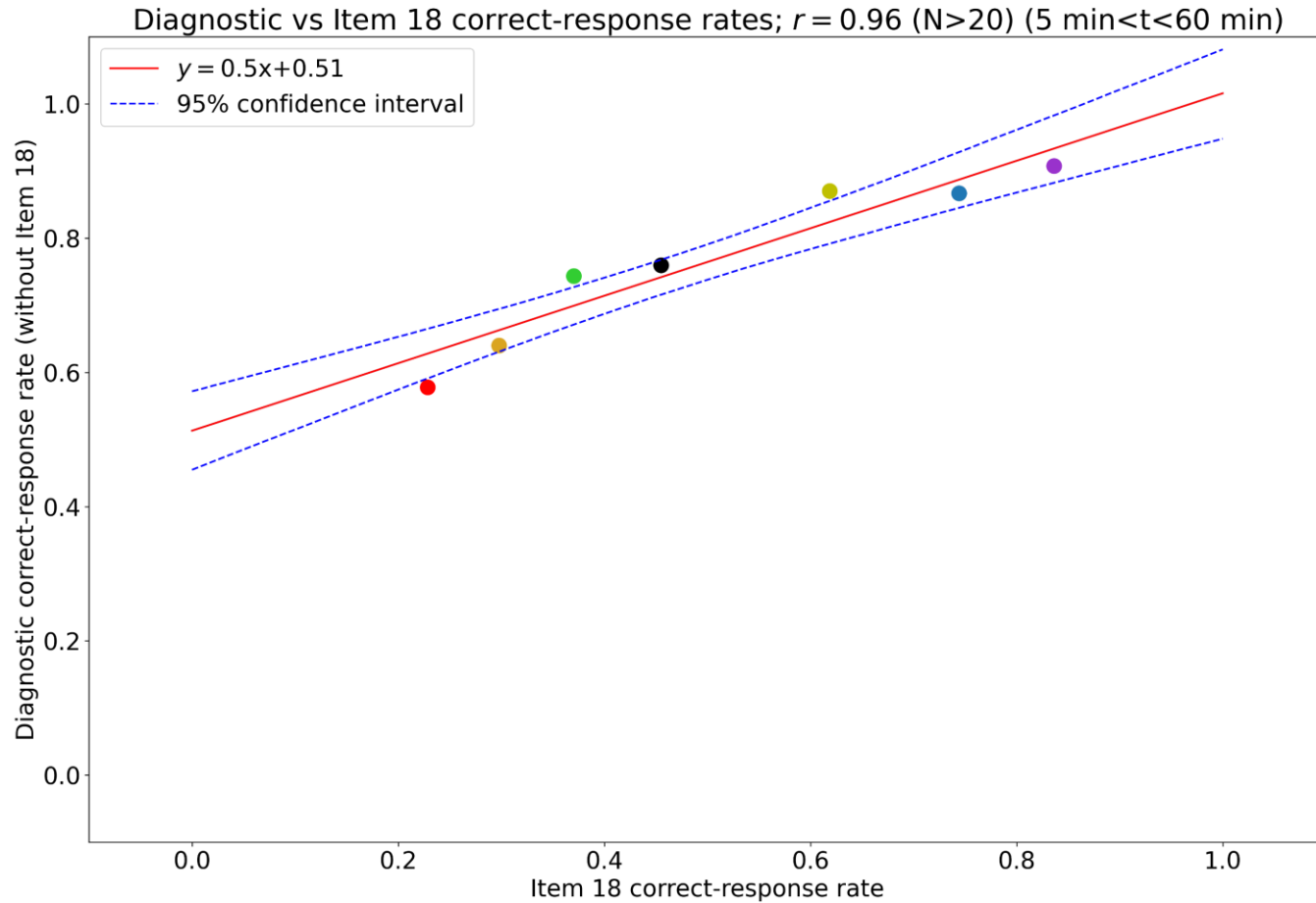
F. $\frac{a}{db}$

H. $\frac{a}{b+d}$

J. $\frac{c}{d} (a-b)$

Online vs hand-written: course-level predictive power

Samples used in fit: written

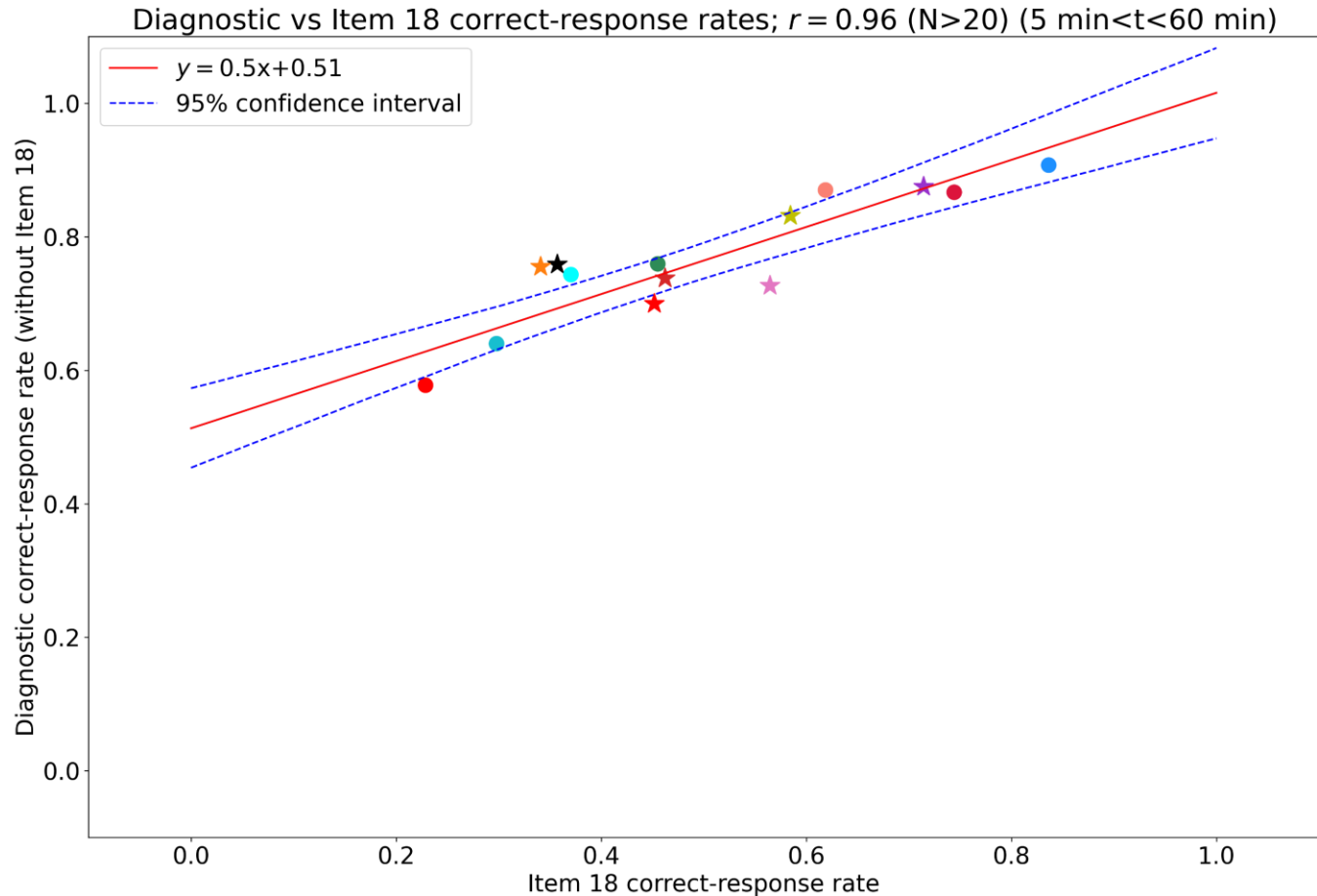


Written only

- written, PHY111, ASU Polytechnic, Pre, Spring, 2020, N=35
- written, PHY111, ASU Tempe, Pre, Spring, 2020, N=47
- written, PHY111, CU, Pre, Fall, 2019, N=167
- written, PHY121, ASU Polytechnic, Pre, Spring, 2020, N=27
- written, PHY121, ASU Tempe, Pre, Spring, 2020, N=173
- written, PHY131, ASU Tempe, Pre, Fall, 2019, N=110
- written, PHY131, ASU Tempe, Pre, Spring, 2020, N=86

Online vs hand-written: course-level predictive power

Samples used in fit: written



Written and online

- ★ online, PHY111, ASU Tempe, Post, Spring, 2021, N=31
- ★ online, PHY112, ASU Tempe, Mid, Spring, 2021, N=126
- ★ online, PHY121, ASU Tempe, Post, Spring, 2021, N=426
- ★ online, PHY131, ASU Tempe, Post, Spring, 2021, N=21
- ★ online, PHY2048, UWF, Post, Spring, 2021, N=88
- ★ online, PHY2048, UWF, Pre, Spring, 2021, N=106
- ★ online, PHY2049, UWF, Pre, Spring, 2021, N=62
- written, PHY111, ASU Polytechnic, Pre, Spring, 2020, N=35
- written, PHY111, ASU Tempe, Pre, Spring, 2020, N=47
- written, PHY111, CU, Pre, Fall, 2019, N=167
- written, PHY121, ASU Polytechnic, Pre, Spring, 2020, N=27
- written, PHY121, ASU Tempe, Pre, Spring, 2020, N=173
- written, PHY131, ASU Tempe, Pre, Fall, 2019, N=110
- written, PHY131, ASU Tempe, Pre, Spring, 2020, N=86

Online vs hand-written: course-level predictive power

Item 6

Solve for θ .

$$\gamma\theta + \eta = \lambda\theta + \omega$$

A. $\frac{\eta + \omega}{\gamma - \lambda}$

C. $\frac{\gamma - \lambda}{\omega - \eta}$

E. $\frac{\eta - \omega}{\gamma\lambda}$

G. $\frac{\omega - \eta}{\gamma - \lambda}$

I. $\frac{\eta - \omega + \gamma}{\lambda}$

B. $\frac{\eta - \omega}{\lambda - \gamma}$

D. $\frac{\lambda - \gamma}{\eta - \omega}$

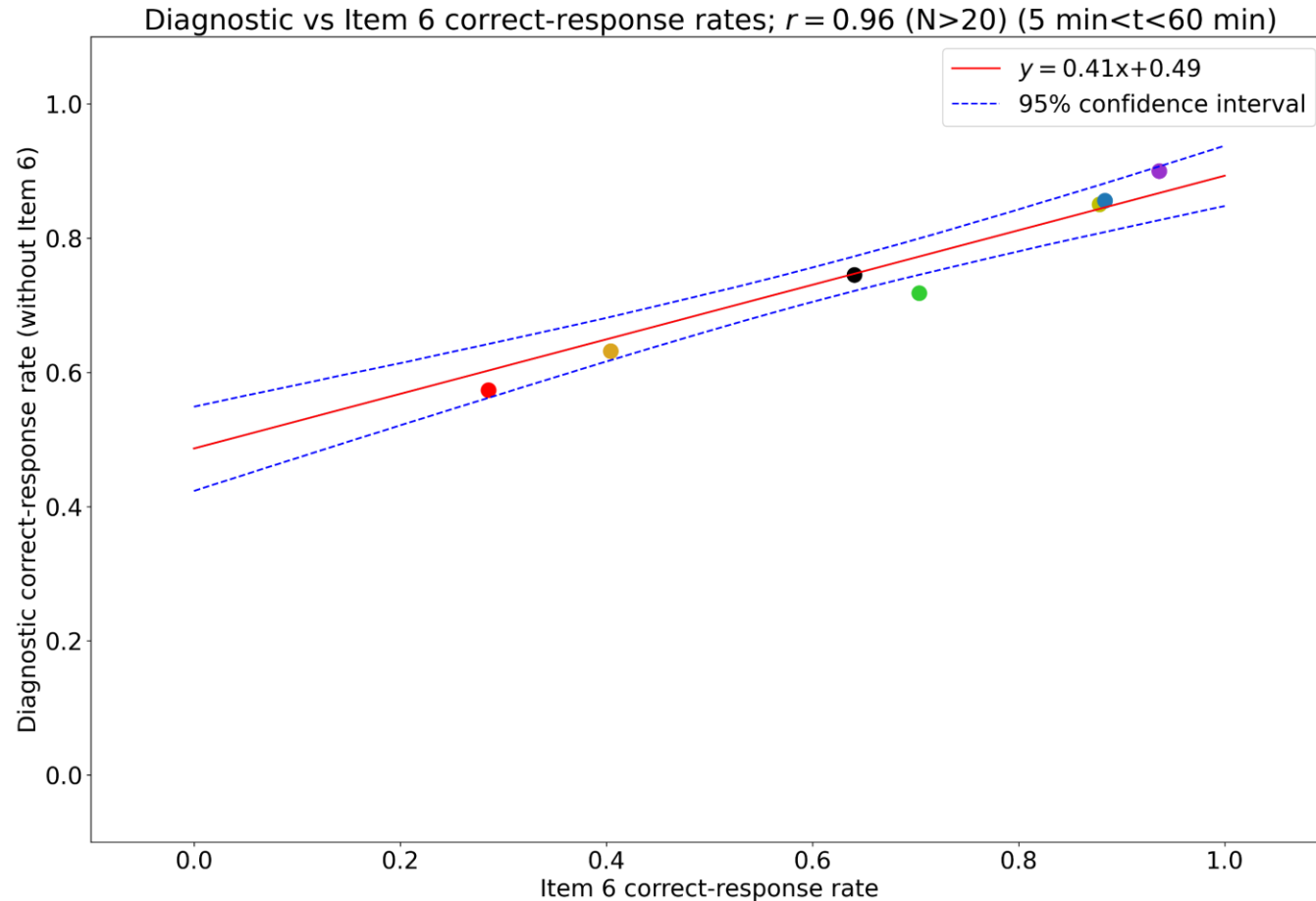
F. $\frac{\omega - \eta}{\gamma\lambda}$

H. $\frac{\omega - \eta}{\gamma + \lambda}$

J. $\frac{\omega - \eta + \lambda}{\gamma}$

Online vs hand-written: course-level predictive power

Samples used in fit: written

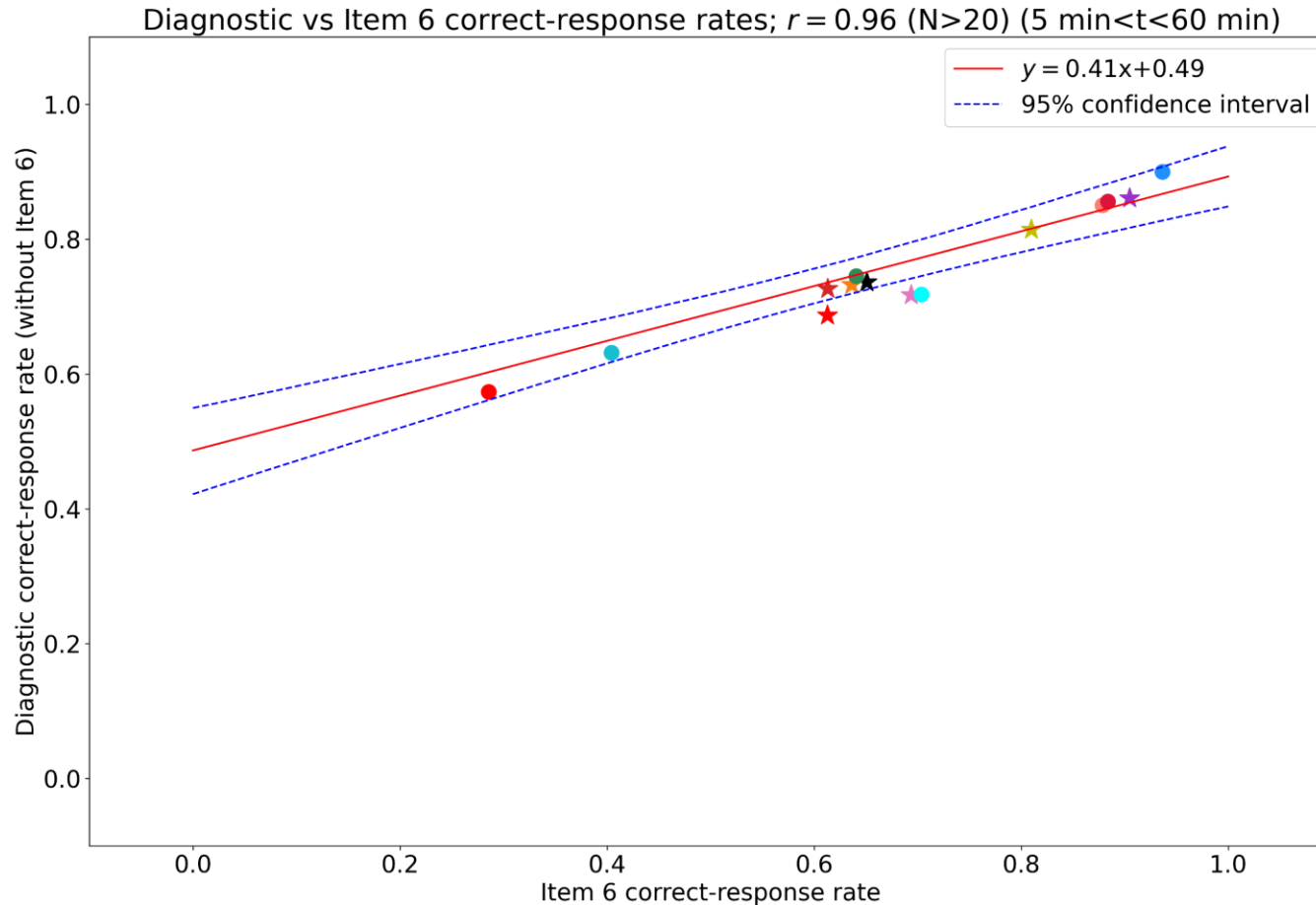


Written only

- written, PHY111, ASU Polytechnic, Pre, Spring, 2020, N=35
- written, PHY111, ASU Tempe, Pre, Spring, 2020, N=47
- written, PHY111, CU, Pre, Fall, 2019, N=167
- written, PHY121, ASU Polytechnic, Pre, Spring, 2020, N=27
- written, PHY121, ASU Tempe, Pre, Spring, 2020, N=173
- written, PHY131, ASU Tempe, Pre, Fall, 2019, N=110
- written, PHY131, ASU Tempe, Pre, Spring, 2020, N=86

Online vs hand-written: course-level predictive power

Samples used in fit: written

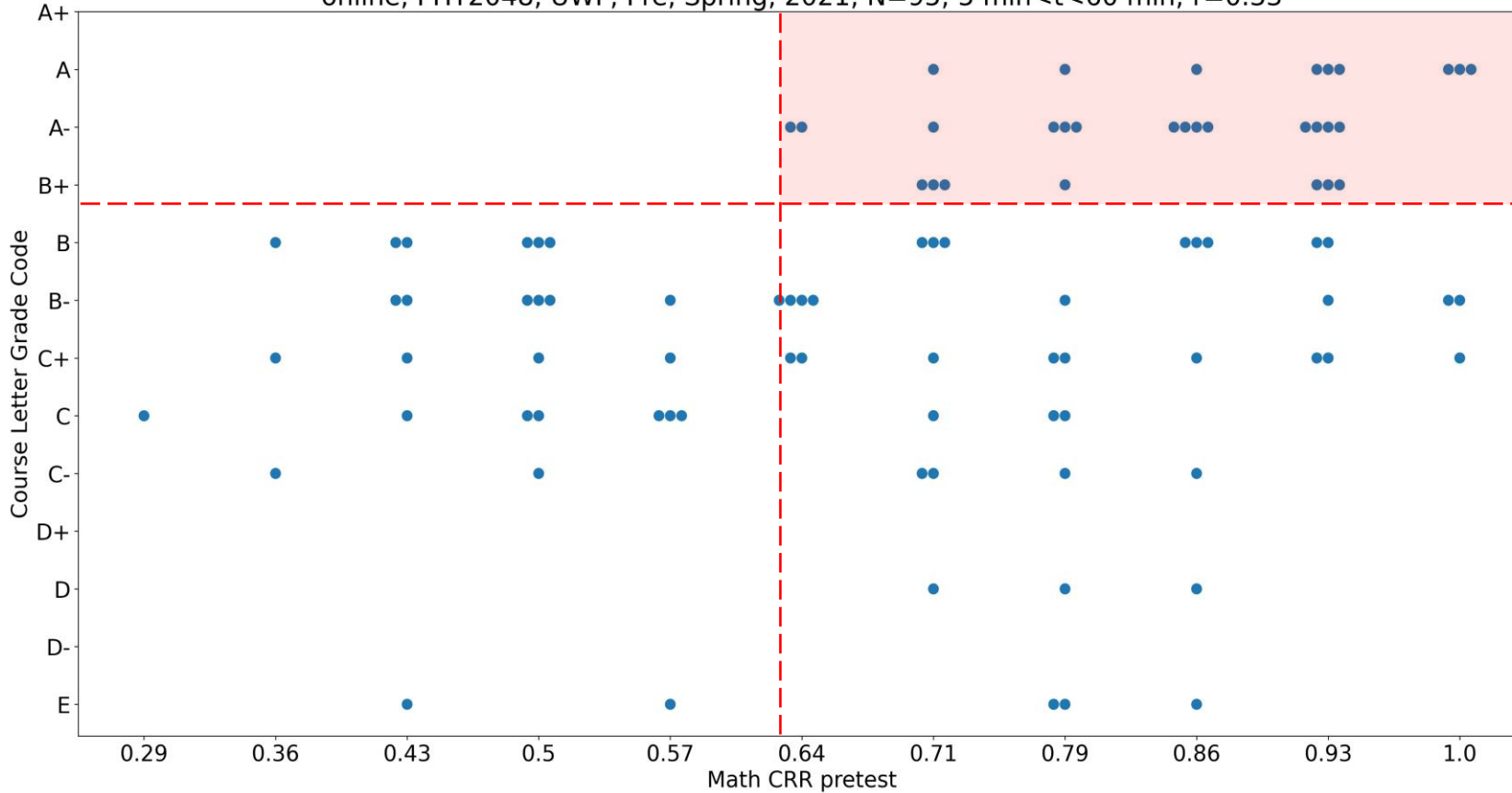


Written and online

- ★ online, PHY111, ASU Tempe, Post, Spring, 2021, N=31
- ★ online, PHY112, ASU Tempe, Mid, Spring, 2021, N=126
- ★ online, PHY121, ASU Tempe, Post, Spring, 2021, N=426
- ★ online, PHY131, ASU Tempe, Post, Spring, 2021, N=21
- ★ online, PHY2048, UWF, Post, Spring, 2021, N=88
- ★ online, PHY2048, UWF, Pre, Spring, 2021, N=106
- ★ online, PHY2049, UWF, Pre, Spring, 2021, N=62
- written, PHY111, ASU Polytechnic, Pre, Spring, 2020, N=35
- written, PHY111, ASU Tempe, Pre, Spring, 2020, N=47
- written, PHY111, CU, Pre, Fall, 2019, N=167
- written, PHY121, ASU Polytechnic, Pre, Spring, 2020, N=27
- written, PHY121, ASU Tempe, Pre, Spring, 2020, N=173
- written, PHY131, ASU Tempe, Pre, Fall, 2019, N=110
- written, PHY131, ASU Tempe, Pre, Spring, 2020, N=86

Course performance vs diagnostic math performance

online, PHY2048, UWF, Pre, Spring, 2021, N=95; 5 min < t < 60 min; r=0.33



- 100% of students who earned a B+ or higher scored at least 64% correct
- Almost half of students with lower grades scored less than 64% correct

Summary

- The online diagnostic appears to be consistent with our hand-written diagnostic in measuring students' mathematical difficulties
- Course-level performance on the math portion of the diagnostic can be accurately predicted using a single math item
- Performance on the math portion of the diagnostic is related to students' final course grade

