Realistic Assessment of Students' Mathematical Preparation in Introductory Physics Courses

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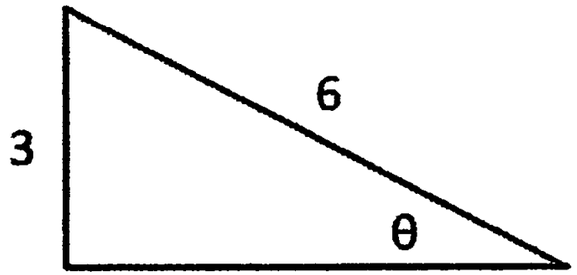
Overview

We have given diagnostic pretests covering pre-college mathematics to over 7000 introductory physics students:

- Results from five campuses at four different state universities were consistent
- Results on an online version are consistent with those on the written version
- High and low scores on the diagnostic are somewhat predictive of course grades

Examples of Test Items

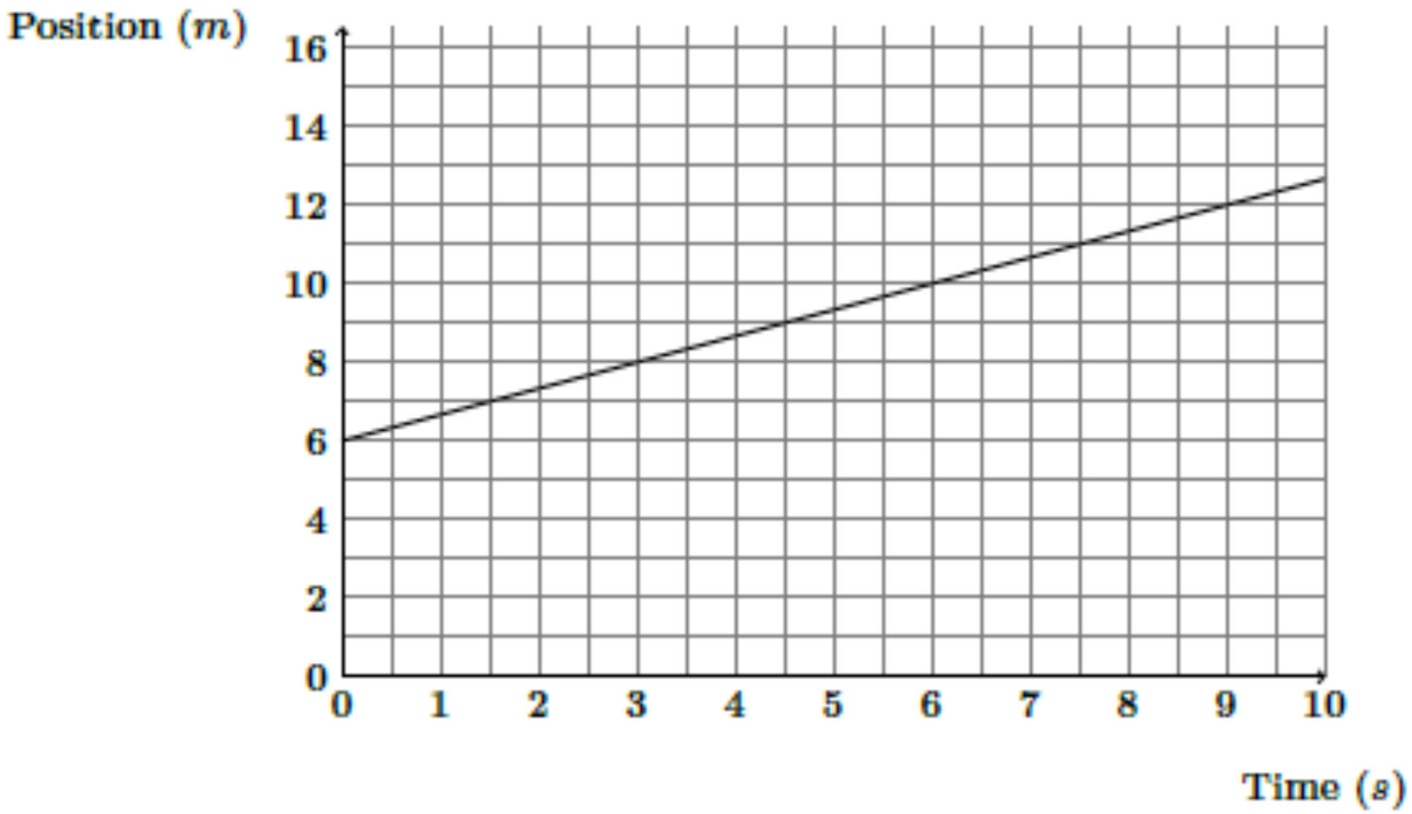
Find Unknown Angle



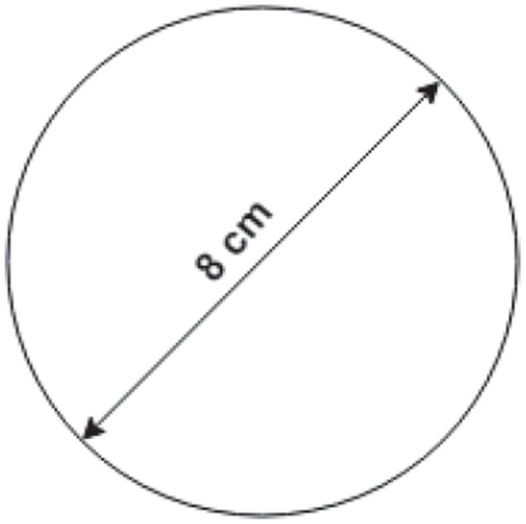
What is the value of θ?

Find Slope of Graph

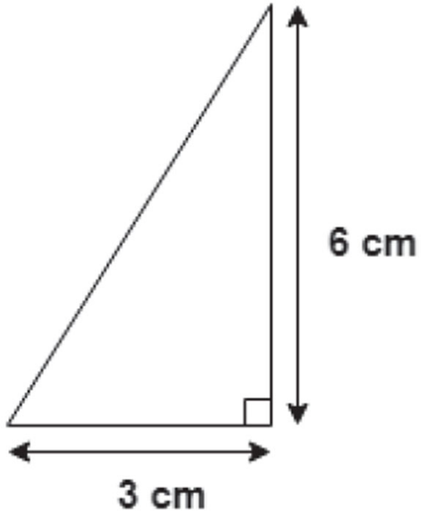
What is the slope of the graph below?



Find Area



(a) Area of the circle =



(b) Area of the triangle =

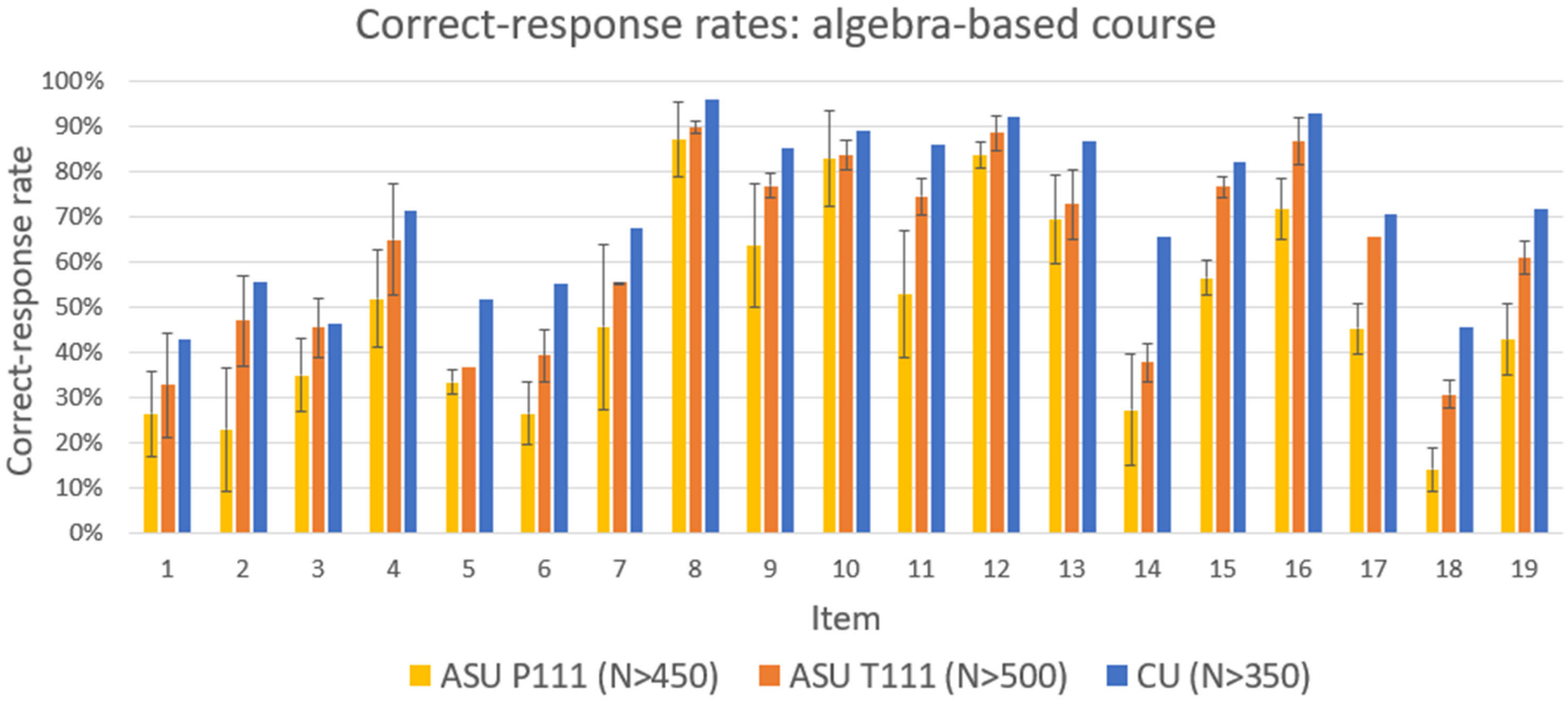
Simultaneous Equations, Symbolic Coefficients

$$cy = dx$$

$$a - y = bx$$

$$x = ?$$

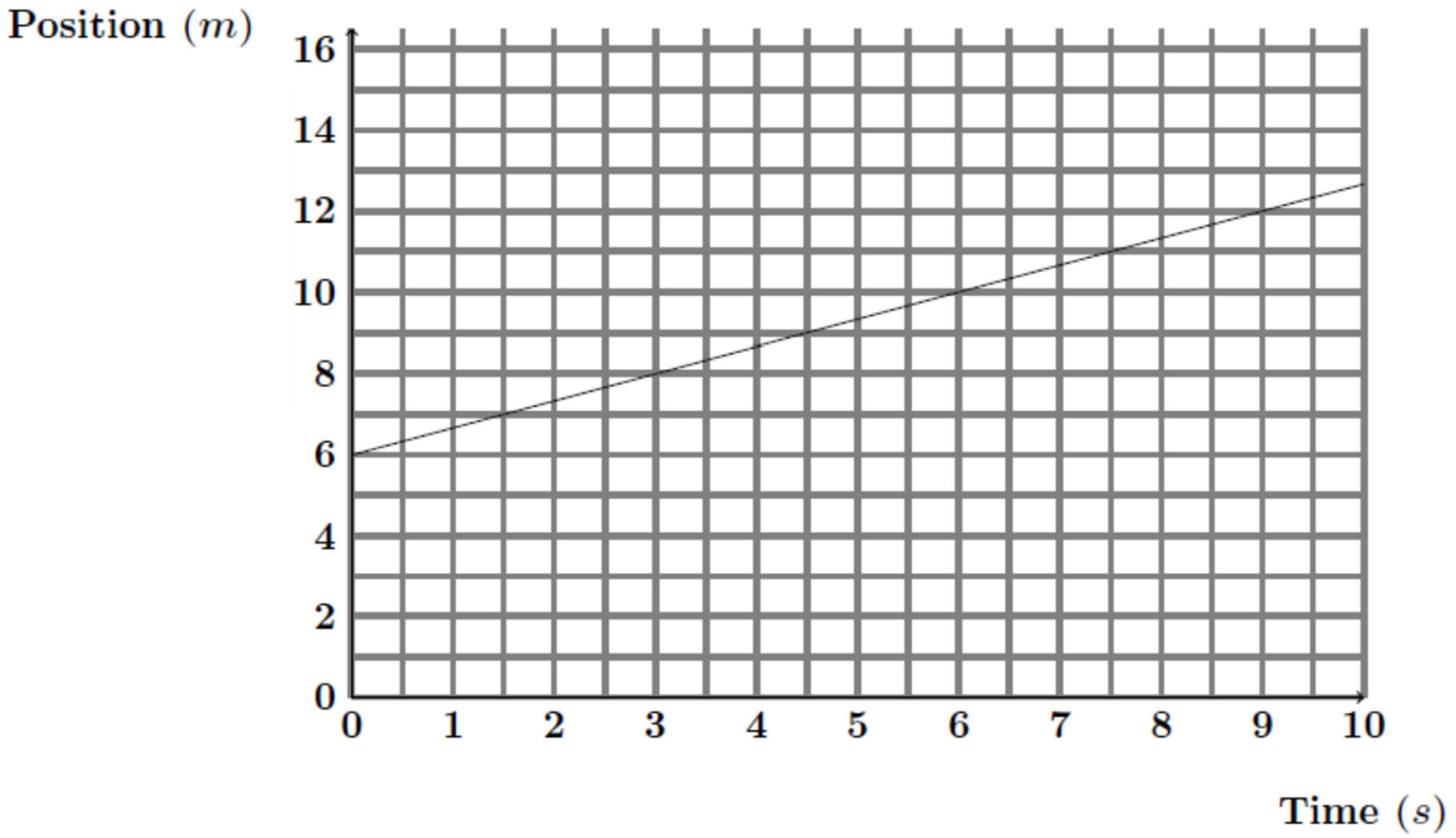
High consistency of results among five campuses at four different universities (three campuses shown below) suggests findings are generalizable



Students show weakness with units and graphing

- Many students ignored graph-axis labels, and provided no or incorrect units for area and velocity.

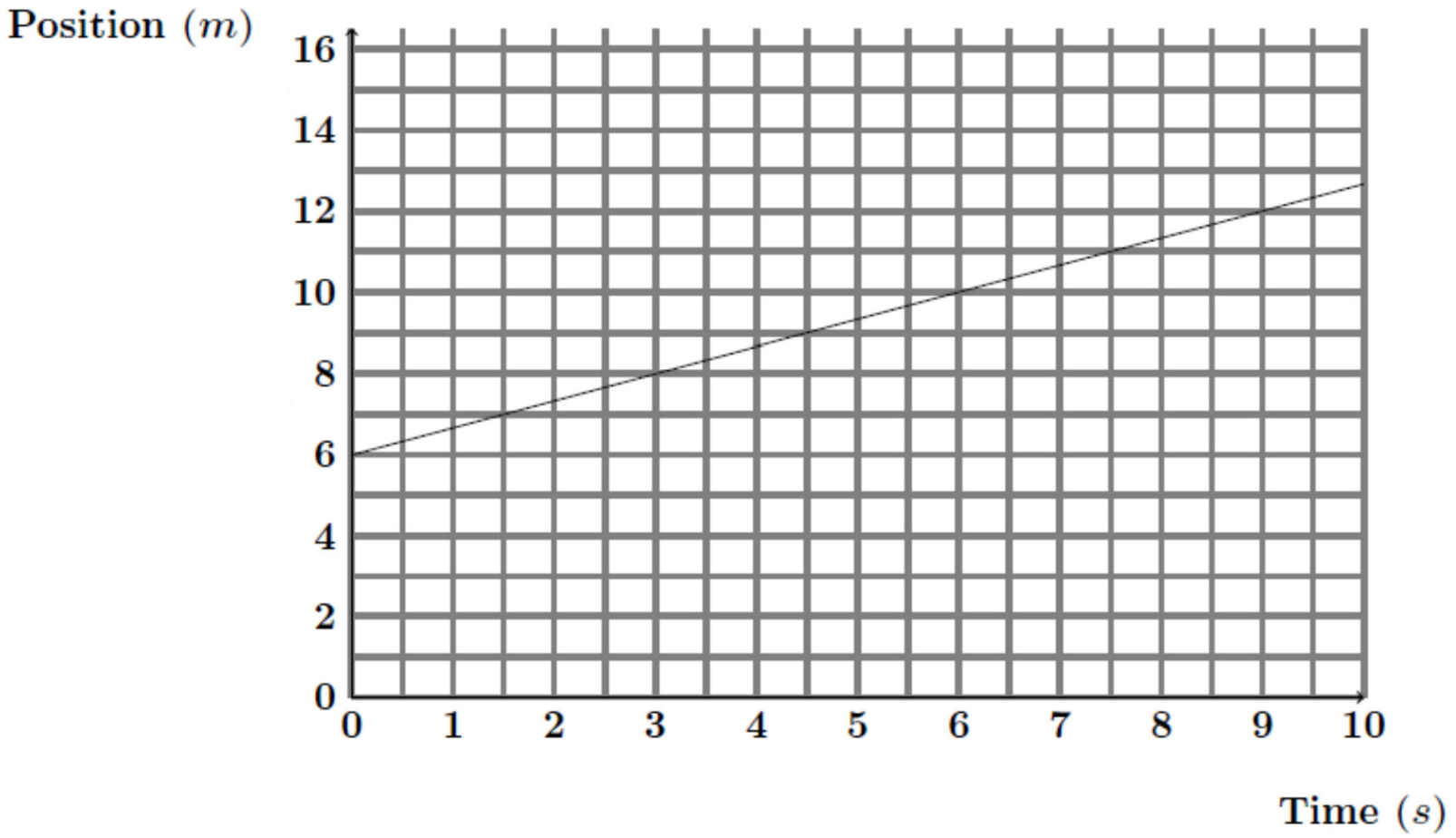
What is the slope of the graph below?



What is the slope of the graph below?

Correct-response rate ($N > 2000$):

30-60%, nearly independent of course or campus



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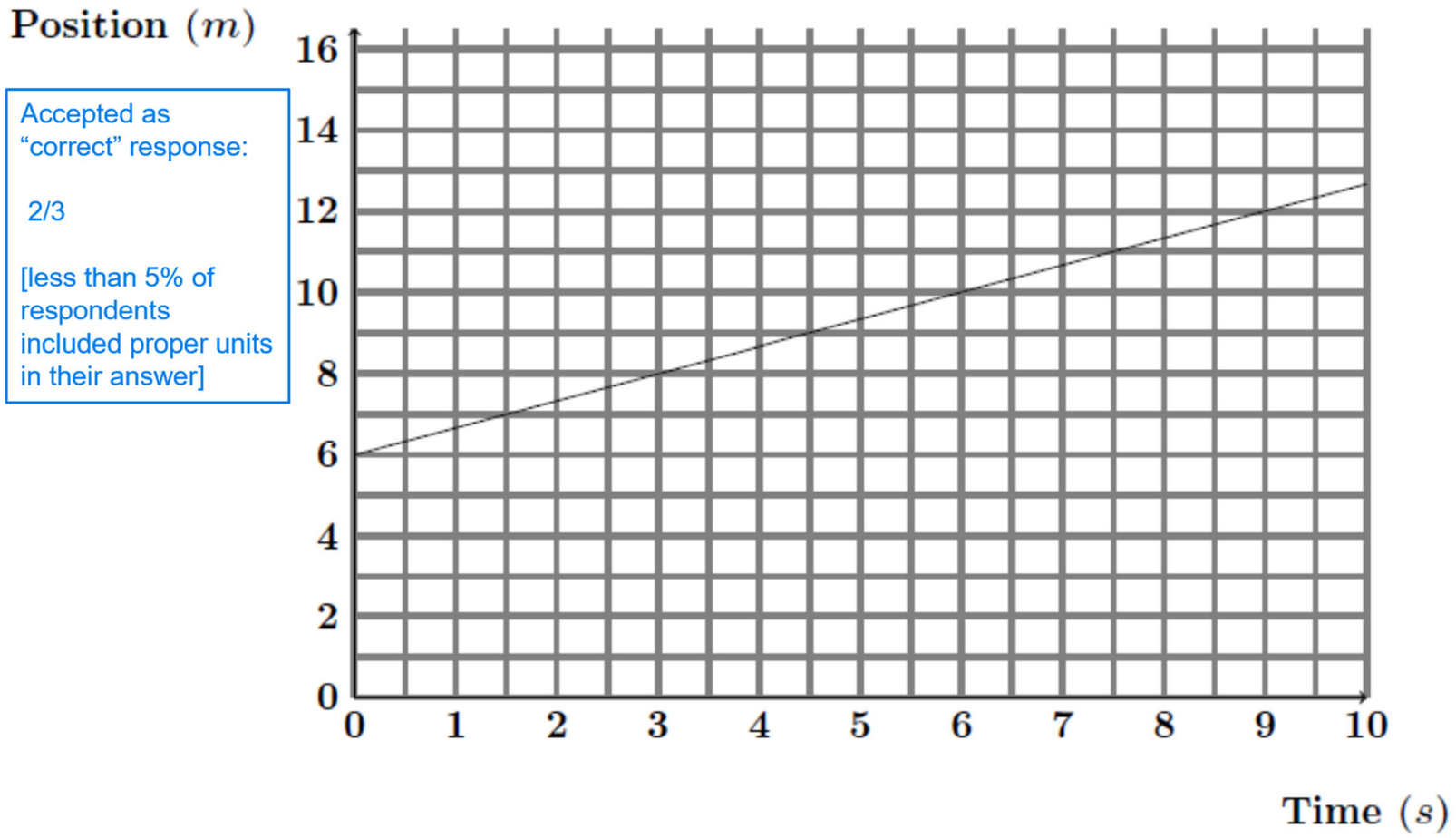
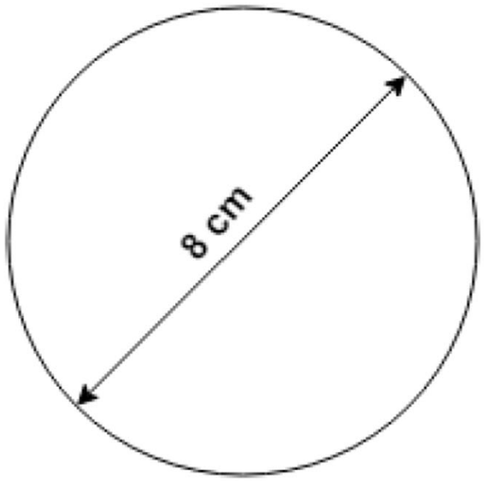


Accepted as "correct" response:

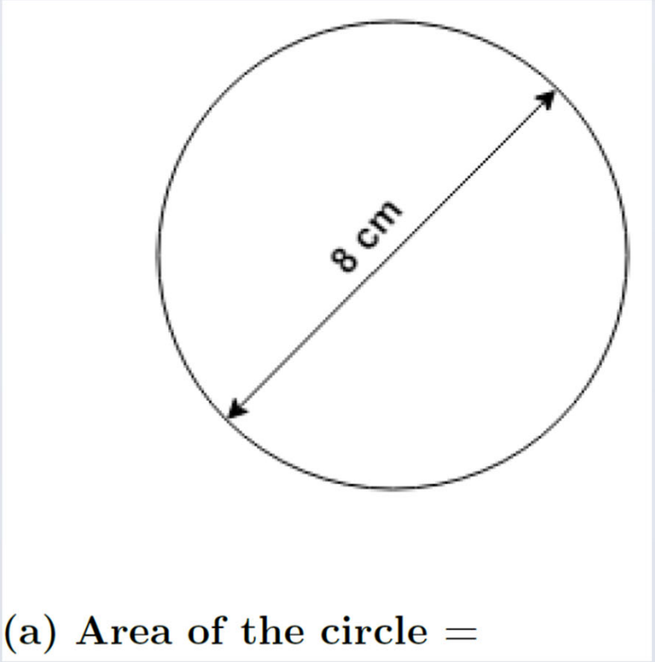
2/3

[less than 5% of respondents included proper units in their answer]

Most common error: *Counting grid squares and ignoring numbers on axes*

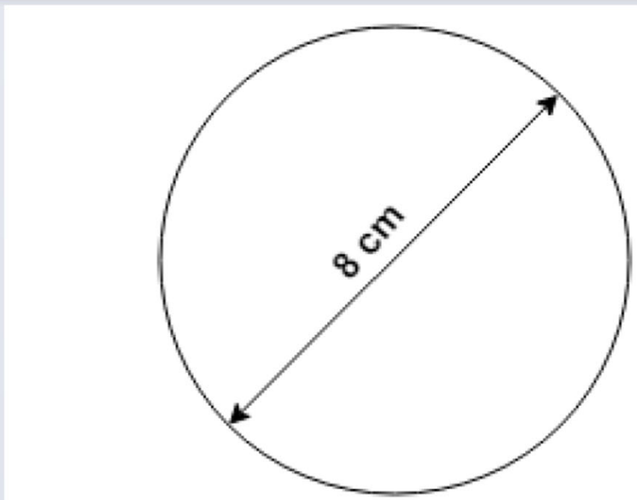


(a) Area of the circle =



(a) Area of the circle =

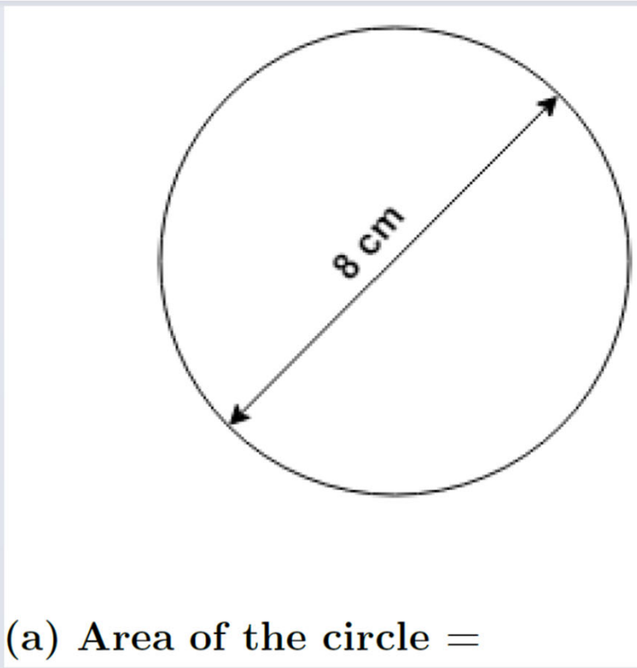
Area of Circle: Algebra- and Calculus-based courses combined, 2018



(a) Area of the circle =

Area of Circle: Algebra- and Calculus-based courses combined, 2018

	N	Numerically correct
ASU-Polytechnic	250	57%
ASU-Tempe	1086	76%

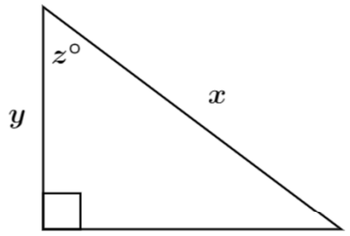


Area of Circle: Algebra- and Calculus-based courses combined, 2018

	N	Numerically correct	Correct with correct units
ASU-Polytechnic	250	57%	29%
ASU-Tempe	1086	76%	45%

On-line Version

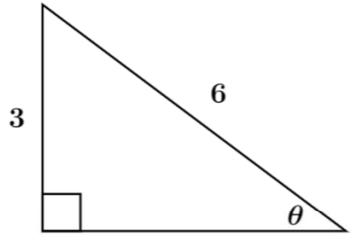
What is the length of side x?



A. $ycos(z°)$	D. $y/cos(z°)$	G. $cos(z°)/y$	J. $\sqrt{y^2+z^2}$
B. $ysin(z°)$	E. $y/sin(z°)$	H. $sin(z°)/y$	K. $\sqrt{z^2-y^2}$
C. $ytan(z°)$	F. $y/tan(z°)$	I. $tan(z°)/y$	L. y/z

(There may be more than one correct answer, but please select only ONE answer.)

What is the value of θ?



A. $cos(3/6)$	D. $cos^{-1}(3/6)$	G. 30°	J. 27°
B. $sin(3/6)$	E. $sin^{-1}(3/6)$	H. 45°	K. 3/6
C. $tan(3/6)$	F. $tan^{-1}(3/6)$	I. 60°	L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

$cos(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$sin(90°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

$tan(0°) = ?$

A. 0 B. 1 C. undefined D. 0.707 E. 0.894

(There may be more than one correct answer, but please select only ONE answer.)

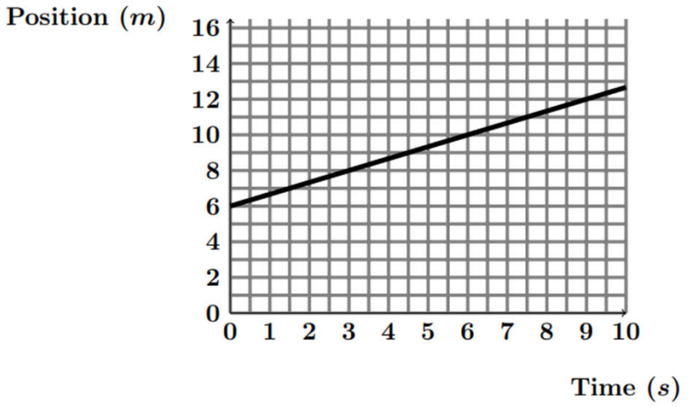
Solve for θ.

$$\gamma\theta + \eta = \lambda\theta + \omega$$

A. $\frac{\eta+\omega}{\gamma-\lambda}$	C. $\frac{\gamma-\lambda}{\omega-\eta}$	E. $\frac{\eta-\omega}{\gamma\lambda}$	G. $\frac{\omega-\eta}{\gamma-\lambda}$	I. $\frac{\eta-\omega+\gamma}{\lambda}$
B. $\frac{\eta-\omega}{\lambda-\gamma}$	D. $\frac{\lambda-\gamma}{\eta-\omega}$	F. $\frac{\omega-\eta}{\gamma\lambda}$	H. $\frac{\omega-\eta}{\gamma+\lambda}$	J. $\frac{\omega-\eta+\lambda}{\gamma}$

(There may be more than one correct answer, but please select only ONE answer.)

What is the slope of the graph below?



A. $\frac{1}{3}$ m/s because the object moves 1 meter in 3 seconds.

B. $\frac{1}{3}$ m/s because the line rises 1 box while it goes 3 boxes in the horizontal direction.

C. $\frac{2}{3}$ m/s because the object moves 2 meters in 3 seconds.

D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

(There may be more than one correct answer, but please select only ONE answer.)

$$\left(\frac{a}{3}\right)^3 = ?$$

A. $\frac{a^3}{3}$ B. $\frac{a}{27}$ C. $\frac{a^3}{27}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{a}{b}\right) = ?$$

A. $\frac{2a}{b}$ B. $\frac{2a}{2b}$ C. $\frac{a}{2b}$

(There may be more than one correct answer, but please select only ONE answer.)

$$\frac{a/b}{c^2/d} = ?$$

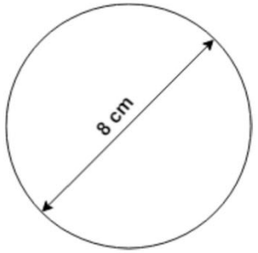
A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

$$2\left(\frac{3}{4}\right) = ?$$

A. $\frac{6}{8}$ B. $\frac{12}{8}$ C. $\frac{3}{8}$ D. $\frac{3}{2}$ E. $\frac{3}{4}$

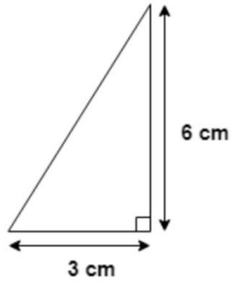
(There may be more than one correct answer, but please select only ONE answer.)



(a) Area of the circle = ?

A. 8π cm^3	F. 8π cm^2	K. 8π cm
B. 16π cm^3	G. 16π cm^2	L. 16π cm
C. 32π cm^3	H. 32π cm^2	M. 32π cm
D. 64π cm^3	I. 64π cm^2	N. 64π cm
E. 128π cm^3	J. 128π cm^2	O. 128π cm

(There may be more than one correct answer, but please select only ONE answer.)



(b) Area of the triangle = ?

A. 4.5 cm^3	F. 4.5 cm^2	K. 4.5 cm
B. 9 cm^3	G. 9 cm^2	L. 9 cm
C. 12 cm^3	H. 12 cm^2	M. 12 cm
D. 18 cm^3	I. 18 cm^2	N. 18 cm
E. 36 cm^3	J. 36 cm^2	O. 36 cm

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

$$\frac{3}{2} = 7x$$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$v^2 = v_0^2 + 2ad$$

$$v_0 = 0$$

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = 60$$

$$\Delta t = 8$$

$$v = 30$$

d =?

A. $d = 30$ B. $d = 60$ C. $d = 120$ D. $d = 240$ E. $d = 480$

(There may be more than one correct answer, but please select only ONE answer.)

$$cy = dx$$

$$a - y = bx$$

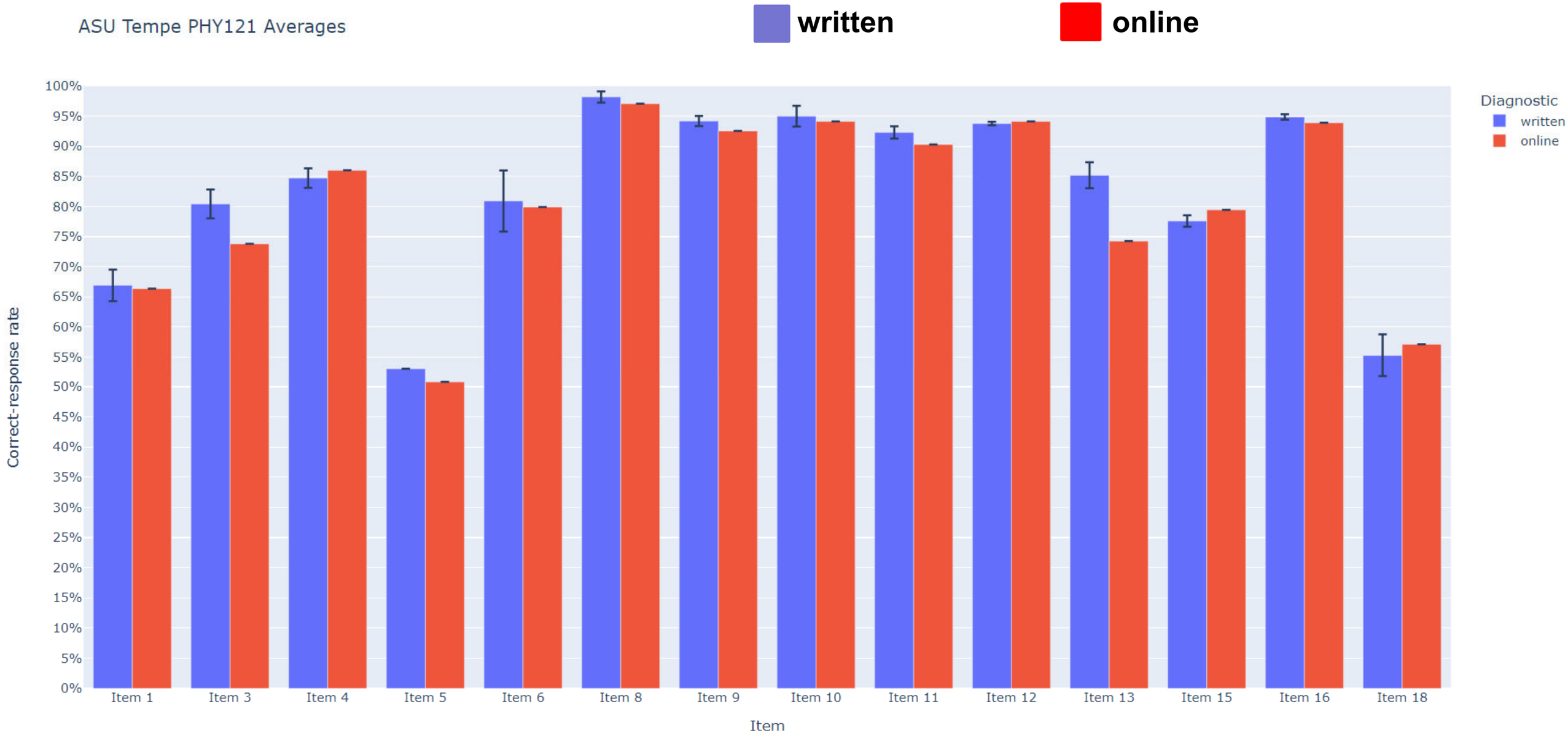
x =?

A. $\frac{ac}{d+b}$ C. $\frac{ac}{bc-d}$ E. $\frac{ac}{db}$ G. $\frac{a}{b+\frac{d}{c}}$ I. $\frac{1}{b}\left(a-\frac{d}{c}\right)$

B. $\frac{ac}{d-b}$ D. $\frac{ac}{bc+d}$ F. $\frac{a}{db}$ H. $\frac{a}{b+d}$ J. $\frac{c}{d}\left(a-b\right)$

(There may be more than one correct answer, but please select only ONE answer.)

On-line and written versions yield consistent results



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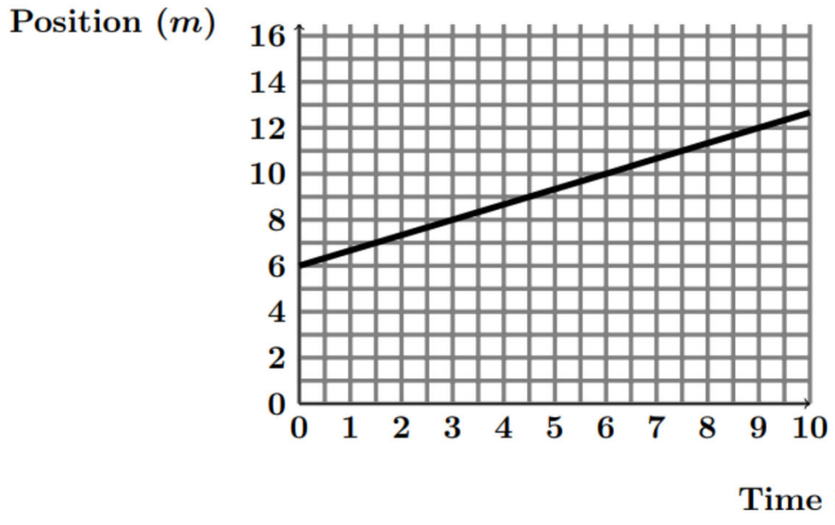

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D. $\frac{2}{3}$ m/s because the line rises 2 boxes while it goes 3 boxes in the horizontal direction.

***N* = 2556**

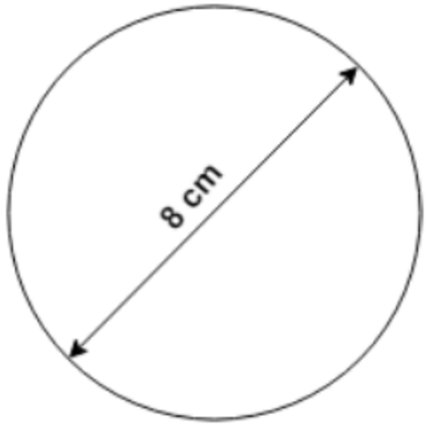
Numerically correct (C or D): 59%

***Actually* correct (C): 48%**

Consistent with results on written version

Most common error: *Counting grid squares and ignoring numbers on axes*

On-line Version:



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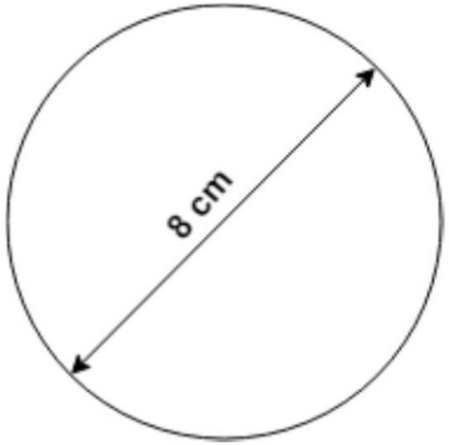
C. 32π cm	H. 32π cm^2	M. 32π cm^3
D. 64π cm	I. 64π cm^2	N. 64π cm^3
E. 128π cm	J. 128π cm^2	O. 128π cm^3

20% did *not* choose cm^2

(*N* = 1252)



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B. 16π cm	G. $\boxed{16\pi \text{ cm}^2}$	L. 16π cm^3
C. 32π cm	H. 32π cm^2	M. 32π cm^3
D. 64π cm	I. 64π cm^2	N. 64π cm^3
E. 128π cm	J. 128π cm^2	O. 128π cm^3

Calculus-based Course, ASU-Tempe (*N* = 430)

G: 68%

B: 10%

L: 2%

Other: 20%

Symbolic notation degrades student performance

- Use of symbols to replace numbers in otherwise identical algebraic equations lowered correct-response rates by ≈25%.

Algebra: Simultaneous Equations (calculus-based course)

$0.5y = 2x$
$78.4 - y = 8x$

[Solve for x]

Numeric Version 79% correct (N = 1043)

Algebra: Simultaneous Equations (calculus-based course)

$0.5y = 2x$
$78.4 - y = 8x$

[Solve for x]

Numeric Version 79% correct (N = 1043)

$cy = dx$
$a - y = bx$

[Solve for x]

Symbolic Version 55% correct (N = 862)

Findings from >70 Interviews: Students make many “careless” errors

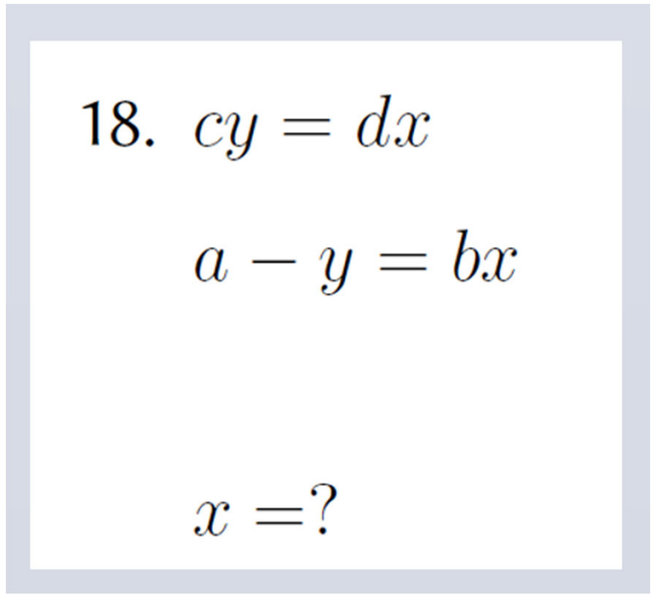
- During interviews, students tended to self-correct approximately 60% of their initial errors, suggesting many errors are “careless.”

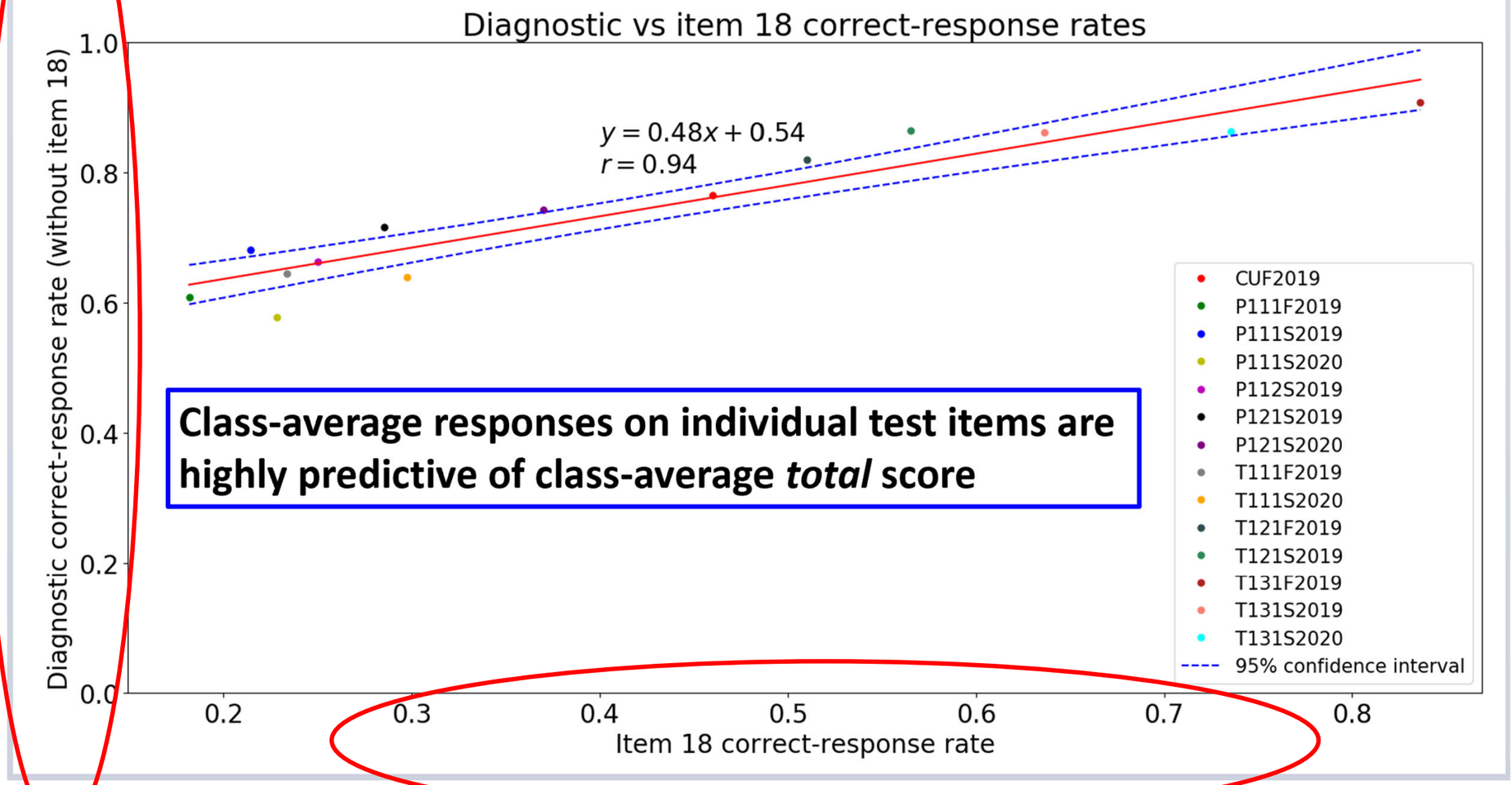
Even single test items are highly predictive

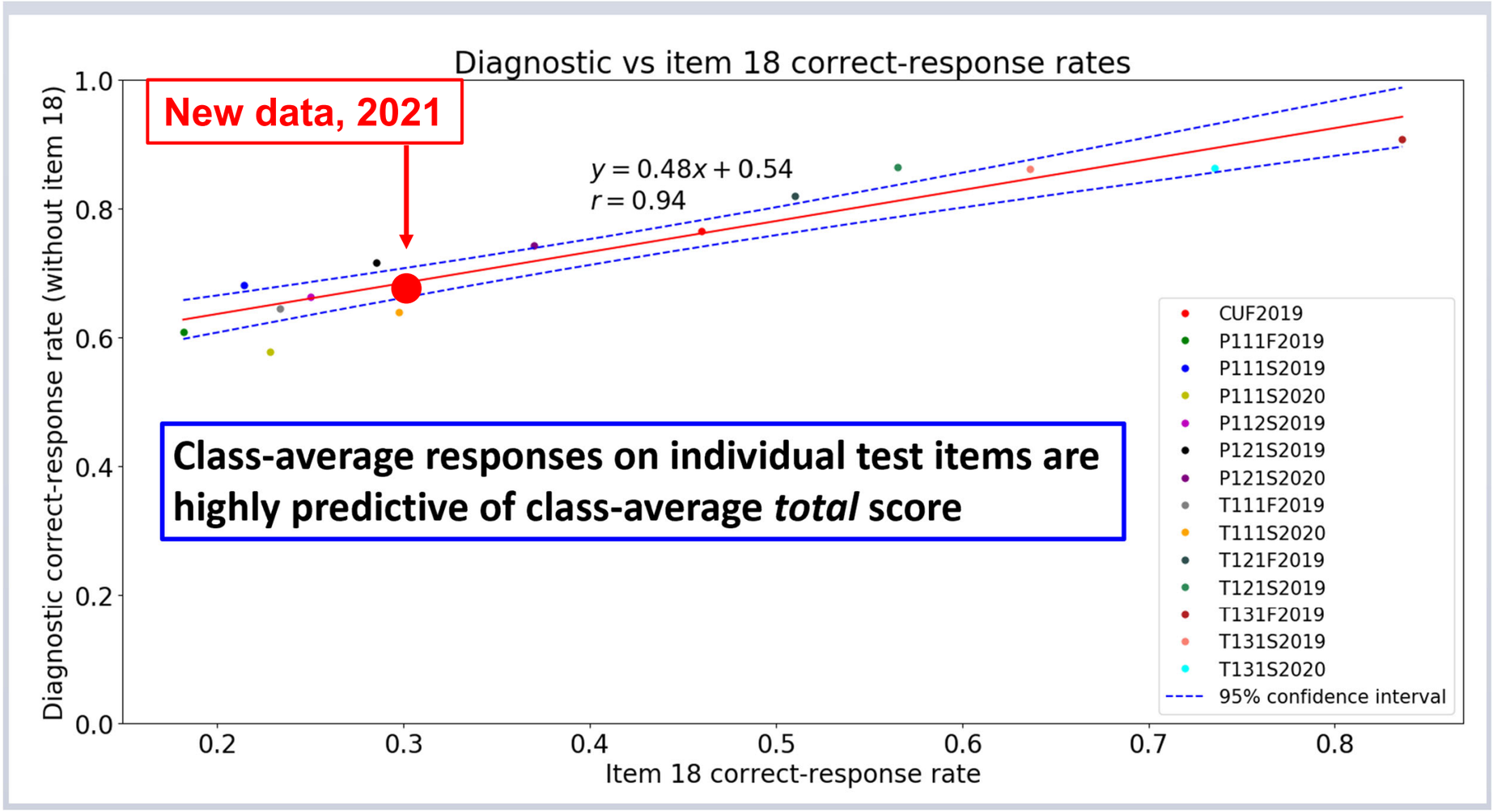
- Performance on **one single diagnostic item** can *accurately* predict class-average score on full 13-item diagnostic

Example:

[#18]




Diagnostic vs item 18 correct-response rates
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
$y = 0.48x + 0.54$
$r = 0.94$
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval
Class-average responses on individual test items are highly predictive of class-average *total* score


Diagnostic vs item 18 correct-response rates
New data, 2021
$y = 0.48x + 0.54$
$r = 0.94$
Class-average responses on individual test items are highly predictive of class-average *total* score
Diagnostic correct-response rate (without item 18)
Item 18 correct-response rate
1.0
0.8
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0.3
0.4
0.5
0.6
0.7
0.8
CUF2019
P111F2019
P111S2019
P111S2020
P112S2019
P121S2019
P121S2020
T111F2019
T111S2020
T121F2019
T121S2019
T131F2019
T131S2019
T131S2020
95% confidence interval

Implication: It may be possible to diagnose the level of students' difficulties with only one or very few mathematics pretest items.

Scores on 3-item Subset: Relation to High Course Grades

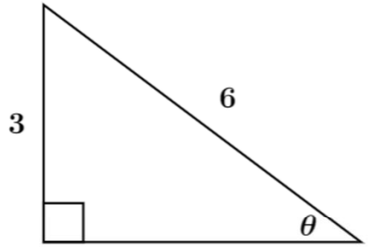
- Can performance on a **3-item subset** predict final course grade?

Example:

[#3, #11, #12]

What is the value of θ?

#3



A. $cos(3/6)$	D. $cos^{-1}(3/6)$	G. $30°$	J. $27°$
B. $sin(3/6)$	E. $sin^{-1}(3/6)$	H. $45°$	K. $3/6$
C. $tan(3/6)$	F. $tan^{-1}(3/6)$	I. $60°$	L. 0.524

(There may be more than one correct answer, but please select only ONE answer.)

Solve for x.

#12

$$\frac{3}{2} = 7x$$

A. $\frac{14}{3}$ B. $\frac{3}{14}$ C. $\frac{21}{2}$ D. $\frac{21}{14}$

(There may be more than one correct answer, but please select only ONE answer.)

$$\frac{a/b}{c^2/d} = ?$$

#11

A. $\frac{ac^2}{bd}$ B. $\frac{ad}{bc^2}$ C. $\frac{bd}{ac^2}$ D. $\frac{bc^2}{ad}$

(There may be more than one correct answer, but please select only ONE answer.)

High Course Grade vs. Subset Score

Course	Campus	*N*	% grade ≥ B+ overall
Alg-1	ASU-P	78	49%

High Course Grade vs. Subset Score

Course	Campus	*N*	% grade ≥ B+ overall	% grade ≥ B+ 3/3	% grade ≥ B+ 0/3 or 1/3	High-grade Ratio 3/3 score vs. 0/3 or 1/3 score
Alg-1	ASU-P	78	49%	68%	37%	1.8

High Course Grade vs. Subset Score

Course	Campus	*N*	% grade ≥ B+ overall	% grade ≥ B+ 3/3	% grade ≥ B+ 0/3 or 1/3	High-grade Ratio 3/3 score vs. 0/3 or 1/3 score
Alg-1	ASU-P	78	49%	68%	37%	1.8
Alg-2	ASU-P	72	44%	54%	30%	1.8
Alg-2	ASU-T	129	74%	75%	68%	1.1
*Calc-1	UWF	103	32%	53%	4%	13.3
Calc-2	UWF	59	58%	70%	56%	1.3

*subset optimized for this course

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Relation Between Scores and Grades

- Performance on **full online diagnostic** can *approximately* predict final course grade

High Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≥ A- overall	% grade ≥ A- score ≥ 81%	% grade ≥ A- score ≤ 57%	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1	ASU-P	78	35%	63%	15%	4.2

High Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≥ A- overall	% grade ≥ A- score ≥ 81%	% grade ≥ A- score ≤ 57%	High-grade Ratio score ≥ 81% vs. score ≤ 57%
Alg-1	ASU-P	78	35%	63%	15%	4.2
Alg-2	ASU-P	72	39%	64%	25%	2.6
Alg-2	ASU-T	129	60%	67%	55%	1.2
Calc-1	UWF	103	22%	40%	0%	“∞”
Calc-2	UWF	59	49%	61%	38%	1.6

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored high on math diagnostic pretest had more “A” course grades than those who scored low

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≤ B- overall	% grade ≤ B- score ≥ 81%	% grade ≤ B- score ≤ 57%	Low-grade Ratio score ≤ 57% vs. score ≥ 81%

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≤ B- overall
Alg-1	ASU-P	78	25%
Alg-2	ASU-P	72	33%

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≤ B- overall	% grade ≤ B- score ≥ 81%	% grade ≤ B- score ≤ 57%	Low-grade Ratio score ≤ 57% vs. score ≥ 81%
Alg-1	ASU-P	78	25%			
Alg-2	ASU-P	72	33%			

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
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Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Low Course Grade vs. Full Diagnostic Score

Course	Campus	*N*	% grade ≤ B- overall	% grade ≤ B- score ≥ 81%	% grade ≤ B- score ≤ 57%	Low-grade Ratio score ≤ 57% vs. score ≥ 81%
Alg-1	ASU-P	78	25%	19%	38%	2.1
Alg-2	ASU-P	72	33%	14%	32%	2.3

Alg-1: Algebra-based course, first semester
Alg-2: Algebra-based course, second semester
Calc-1: Calculus-based course, first semester
Calc-2: Calculus-based course, second semester

ASU-P: Arizona State University, Polytechnic campus
ASU-T: Arizona State University, Tempe campus
UWF: University of West Florida

Students who scored low on math diagnostic pretest had more "C" course grades than those who scored high

Summary

- Instructors should be wary of assumptions about students' mathematics preparation before making assessments
- Pre-instruction performance on a brief mathematics diagnostic may provide indications of students at risk