Student concepts of gravity

by

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CHAPTER 1. INTRODUCTION

Students in introductory physics courses often have difficulty gaining a robust understanding of common physical principles. A student develops an understanding of how the world works well before he or she enters a classroom. This student's existing model of how things work affects what he or she can learn from a physics class. McDermott[1] and Redish[2] note that the student's mind is not a "blank slate." Students arrive in physics classes with notions, ideas, and models in their minds. Learning does not simply mean the acquisition of a new batch of rules and ideas; it also involves reconciling this new information with the students' pre-existing ideas. To teach students effectively, it is important to first understand what those existing ideas are. This constructivist epistemology provides a basis for understanding why students often have difficulties with physics topics such as gravity.

Physics education research has blossomed in the past several years. This research comes in many varieties: theories of learning, investigation of student concepts and of student attitudes toward physics, factors influencing physics learning, instructional methods, and so on. Research into concepts held by students regarding many topics in physics is extensive. Electrostatics, optics, and electric current are a few examples of such topics. However, there is a surprising lack of research on student concepts of gravity, especially concerning students at the college level. Most work in this area focused on elementary- and middle-school students. The lack of published work on students' concepts of gravity raises a red flag indicating a need for this research.

Studying how concepts of gravity are learned has other benefits. The force of gravity exerted on one object by another depends on the inverse-square of the distance separating them. In electrostatics, Coulomb's law describing the electric force between two charged particles has this same inverse-square distance dependence. Thus, work done studying the learning of the inverse-square law in the context of gravity can be of interest to those researching the same thing in the context of electrostatics as well.

This thesis is organized as follows. Chapter 2 reviews the literature regarding concepts of gravity for a wide range of students. Chapter 3 details the instruments and methods used to collect first-semester and second-semester data on students' concepts of gravity. This includes free-response and multiple-choice diagnostic instruments as well as student interviews. The data on student concepts and misconceptions of gravity are presented in Chapter 4. This chapter is divided into four major subject areas: (1) direction and superposition of gravitational forces, (2) Newton's third law in the context of gravity, (3) Newton's law of gravitation, and (4) the universality of gravity. For each of these four subject areas, a number of questions were presented to introductory physics students to probe their understanding of the relevant concepts. Data representing the responses to these questions (both written and oral) from students in both calculus-based and algebra-based physics classes are presented in this chapter. In chapter 5 there is a discussion of new instructional materials developed to teach some facets of the concept of gravity. These materials are in the form of printed worksheets, and were designed to address some of the learning difficulties discussed in Chapter 4. A discussion of the instructional effectiveness of these materials is presented, including the question of whether this form of instruction has benefits over traditional instruction on gravity. Conclusions of this work are presented in chapter 6, and diagnostics, worksheets, and interview data can be found in the appendices.

CHAPTER 2. LITERATURE REVIEW

As mentioned in the introduction, there is comparatively little published research on student concepts of gravity. The majority of existing literature has been published by researchers with backgrounds in science education rather than in physics. Their research in most cases has focused on pre-college students, from young children to high-school students. In most of the literature, there is an emphasis on uncovering and identifying student misconceptions, though not necessarily exploring the prevalence of these misconceptions. This is most common in papers presenting only interview transcripts of small sample sizes. A general difficulty with this style of research is that it does not provide the reader with the extent of the student learning difficulties that are identified.

A number of studies have been performed to gain a better understanding of how students think about gravity and the Earth. Many of these studies[3-10] are either only marginally related to the issues discussed in this paper or are similar in nature to other work discussed, and will not be further cited.

Several studies[11-15] have been conducted investigating children's concepts of the Earth and their implications on students' thoughts about gravity. Nussbaum and Novak[11] suggest a scheme of five notions, beginning with the most egocentric: a flat Earth and no concept of space. The notions gradually become more sophisticated: the Earth is a ball in space, and we live on the flat part inside the ball; the Earth is a ball in space, and we live on top of the ball; the Earth is a ball where people live all around the ball, and objects either fall to the surface of the Earth or toward the bottom of the ball; finally, reaching the notion of a round Earth where objects fall toward the center of the Earth. The research was initially done with Israeli second-grade students, but followed up later by Mali and Howe[12], by Sneider and Pulos[13], and by Nussbaum[14] with students at more advanced grade levels. In interviews, students were asked a series of questions regarding the nature of the Earth, including how an object might fall in those situations. For example, one interview question posed by Nussbaum and Novak[11] asks what would happen to a rock if dropped through a hole that went all the way through the Earth. All researchers find evidence supporting a progression of conceptual development towards the more scientific view with advance in age and grade level. The development process tends to start with a large majority of the students using the most egocentric model (i.e. that the world is flat and everything falls down toward the surface) and gradually changing to the scientifically compatible one (a round Earth which pulls objects toward its center.) The student responses to individual survey questions are not stated in the paper; rather, the interview questions used are intended to allow the child to be classified into one of the five pre-set categories. One danger of this approach is that it can potentially obscure the actual responses of the students in an effort to fit them into a pre-existing classification system.

Researchers have also asked students about the presence of gravity in a variety of environments and situations. Watts and Zylbersztajn[16] surveyed a set of 125 British 14-year-old students regarding their concepts of force. Of particular interest is a question in which an astronaut stands on the moon's surface and releases a spanner (a wrench) from one hand. When asked to tell what would happen, nearly 80% of the students replied that the spanner would remain in place or float away. Explanations commonly referred to the moon having no gravity or no atmosphere, and thus concluding that no force would be exerted on the spanner. Consistent results are found for Italian middle school students aged 12-13[17] and

Canadian grade-9 students[18]. Noce et al.[19] also finds that this question is still of significant difficulty for students in advanced grades as well as for college students, elementary teachers, and adults. Though the final-year scientific secondary-school students performed the best of any of the groups surveyed, their performance (about 50% correct answers) indicates that the concept is still not well learned. Noce analyzes students' understanding of gravity by reporting the number of students in each of four classifications: the force of gravity is (1) a force which belongs to the Earth, (2) an 'air,' (3) a force defined by its effects, or (4) defined in terms of effects in relation to a cause. While some of the questions asked by Noce are similar to questions asked by Watts and Zylbersztajn, Noce does not provide student responses to individual pencil-and-paper questions. This unfortunately makes it impossible to compare student performance on those questions.

Other studies have focused on whether or not gravity is present in given environments such as on the moon, orbiting with a satellite or spaceship, or underwater. Several researchers address this issue either as a central focus or as one of a number of issues. Ameh[20] presents interview transcripts in which students express several misconceptions about the existence of gravity on the moon. These misconceptions include the idea that there is no gravity or very little gravity on the moon due to it being far away from Earth, or due to the lack of air on the moon. Another belief expressed is that an object has no weight when it is on the moon. Stead[21] and Galili[22] present students' ideas regarding gravity or the lack thereof in an orbiting spacecraft, in space, and on the moon as well. In particular, Stead asks students in 3rd-7th grades whether or not there is gravity present at a given location. The students perform gradually more favorably with advancing grade level. When asked about gravity at a skydiver's location falling from a plane, 26% of students at the 3rd grade level (N

= 257) answer favorably that the gravity is present and is about the same as on the ground. At the 7th grade level (N=74), 65% answer favorably. This trend holds when the students are asked about a spaceman near a satellite, with 19% of 3rd grade students (N=253) answering favorably that there is gravity up where the spaceman is, while 65% of 7th grade students (N=75) answer favorably. In recent work, Sharma et al.[23] questioned Sydney University students about the presence of gravity in an orbiting spaceship and categorized student responses into groups who state gravity is zero (about 50% of students), approximately zero (about 10%), or significant in the spaceship (about 30%). Galili[24, 25] also addresses the issue of the definition of weight in understanding tides and "weightlessness."

A small but growing set of reports in the literature[23-30] addresses gravity directly in physics contexts. Several projects report students being questioned about whether a planet's gravity is determined by variables such as a planet's mass, proximity to the Sun[26-27], or presence of air[28]. Piburn[27] asks students about the difficulty a spaceship would have taking off from a planet of various sizes, densities, and distances from the Sun. The complexity added in introducing density as a factor makes it difficult to judge whether the proximity to the Sun is a dominant factor for these questions.

One of the four major topics discussed in Chapter 4 is the direction and superposition of gravitational forces. Few published works address this issue explicitly, making direction and superposition an attractive subject for study. Gunstone and White[29] consider a question where students predict the movement of a spring scale needle when the scale, holding a bucket of sand, is moved from the classroom to the top of Mt. Everest. Just 29% of first year university students (N=458) correctly predict no noticeable shift. This reinforces his idea that while it is widely understood that gravity decreases with height, students are often naïve regarding the scale of the reduction. Galili[37] takes this one step farther in a question posed to students regarding the effect of the Moon on weighing a box. 68% of students in grades 9-10 (N=34) responded that the Moon will cause a change in the weighing results. Slightly more advanced students (grades 11-12, teachers college, and special one-year preparatory programs for college; N=135) answered similarly, with 76% of them indicating an effect by the Moon. In both groups, students tried to summarize the attractions of the box to the Moon and Earth with vectors. While few studies have addressed student understanding of force direction and application of superposition in gravity contexts, Rainson et al.[31] discuss these issues at a more sophisticated level for electrical forces

There appears to be no discussion in the literature regarding Newton’s third law specifically in the context of gravity, nor on student understanding of Newton’s law of gravitation. Again, the lack of study of student ideas of these concepts in this context makes them attractive areas for study. Learning difficulties with third-law issues[37-41] have been studied extensively in other contexts, too many to provide a complete list here. It is reasonable to expect similar difficulties in using gravity as the context for third-law questions. Meltzer[32] notes that students do not necessarily perform equally well on questions posed using different representations (*Verbal, Diagrammatic, Mathematical/Symbolic,* or *Graphical*). The most effective way to identify if a student’s understanding of a concept is robust is to use multiple representations whenever possible. This is examined for some of the third-law questions in Chapter 4.

Students’ understanding of the universality of gravity is the area of research addressed in this thesis that is most well documented in the literature. These studies[17-21, 23] deal with student concepts of the presence or absence of gravity in space, on the Moon, or during free fall. Not all of the areas we focus on in this thesis can be compared to existing data in the literature, but the prior research into the universality of gravity allows such comparisons to be made.

CHAPTER 3. METHODS AND MATERIALS

The primary methods used for the collection of data on first-semester student concepts were a multiple-choice diagnostic instrument first developed by David Meltzer at Southeastern Louisiana University and a free-response diagnostic instrument developed by the author. Both of these diagnostics targeted college students in introductory physics courses. In addition, these diagnostics are also accessible to college students in introductory astronomy courses.

Free-Response Diagnostic

The free response diagnostic instrument used in this work (Appendix A) consists of ten questions. Some of the questions contain multiple parts. The items are a variety of open-ended free-response, sketching, and multiple-choice questions. All items on the diagnostic require "original" responses from the student; depending on the question, the student must either sketch arrows or explain reasoning for a certain answer.

The questions cover a variety of topics related to gravity, including the direction of gravitational force, superposition of multiple gravitational forces, Newton's third law in the context of gravity, Newton's law of gravitation, and the range of applicability or universality of gravity.

A useful advantage of the free-response format is its ability to allow students to answer in their own words. Multiple-choice questions lack this option. Although some of the free-response questions do list choices for the students, the students are also asked to explain their answers. These explanations provide additional insight into the students' understanding.

The free-response diagnostic is designed to elicit student misconceptions, making it particularly valuable as a pre-instruction diagnostic. However, it can also be used as a post-instruction diagnostic.

The initial development of the free-response diagnostic began during the Spring 1999 semester. It was initially tested on a group of community college students in a general education astronomy course. While this sample was small (N=21), it provided an opportunity to assess the utility of the diagnostic. Student responses and comments were used to modify the diagnostic to include facets of the concepts that had not yet been incorporated. A prime example of this is free-response #2, describing the forces an asteroid exerts on the Earth. The original options for the value of the asteroid's gravitational force on the Earth were "greater than," "less than," and "equal to" the force exerted by the Earth on the asteroid. However, the response that the asteroid exerts no force at all on the Earth had not been included until that explanation was given by some of the community college students.

Multiple-Choice Diagnostic

The multiple-choice diagnostic, which can be found in Appendix B, consists of eleven questions. It is a predecessor to the free-response diagnostic, and covers topics similar to those of the free-response diagnostic. Like the free-response diagnostic, it assesses students' concepts of gravity. The diagnostic also uses multiple representations to ask similar questions about the same topic in different visual formats, such as text, vectors, and diagrams. A major advantage of the multiple-choice diagnostic is the speed and ease with which the data can be compiled and processed. The trade-off is the lack of richness inherent in student explanations.

Administration of the Diagnostics

The multiple-choice and free-response diagnostics have been given to a total of over 2000 students, primarily in algebra-based and calculus-based introductory physics courses at Iowa State University. I will focus on these courses in this thesis. Course numbers for the first- and second-semester courses in algebra-based physics at Iowa State are Physics 111 and 112, respectively. Likewise, the first- and second-semester calculus-based course numbers are Physics 221 and 222. The algebra- and calculus-based courses will be referred to interchangeably with their course numbers throughout the remainder of the text. The administration of most of these diagnostics took place during the Summer 1999, Fall 1999, and Spring 2000 semesters. With the exception of the Fall 1998 and Fall 1999 Physics 112 courses, the introductory physics sequences are traditional (lecture, laboratory, and recitation) two-semester sequences both for the algebra-based and calculus-based students. The fall-semester Physics 112 course is taught using a research-based instructional method[33] making use of "interactive engagement."[34]

Since gravity concepts are typically taught in the first semester of introductory physics at Iowa State, we considered the second-semester physics students to be a fair approximation of a “post-instruction” group having already experienced traditional physics instruction on gravity.

A relatively small number of diagnostics ($20 < N < 50$ per class) were administered to introductory physics classes prior to the Fall 1999 semester. During the Fall 1999 and Spring

2000 semesters, diagnostic tests were administered to a large number of students in the introductory physics and astronomy classes at Iowa State. Large samples (up to N=546 per class) were compiled for students in first-semester algebra-based introductory physics as well as in first- and second-semester calculus-based introductory physics.

The diagnostics were given to students during the first week of class, normally during their first lecture or first recitation period. For each course during a given semester, only one of the two diagnostics was administered. Students were allotted approximately 15 minutes to work on the multiple-choice diagnostic and approximately 20 minutes on the free-response diagnostic. In all cases, the students worked on the diagnostics individually.

On the diagnostics (most commonly on the free-response diagnostic), some students left individual questions blank. A blank response could mean any number of things: the student did not have time to answer the question, the student did not know the answer, or the question was too confusing to answer. Such blank responses are left in the overall samples and designated “no response.” Thus, the percentages of the non-blank responses given by students for a particular question may have been slightly higher if all students had answered that question.

Post-instruction Interviews

As an additional probe of students’ thinking, post-interviews with 28 students in the Physics 221 course were conducted and recorded. These interviews took place at the end of the semester, several weeks after all instruction on gravity had taken place. Eighteen of these interviews were with students whose instruction on gravity was completely traditional, while ten interviews were with students whose recitation instruction on gravity involved worksheets developed by the author (described later in this chapter and in Chapter 5). The average course grade for the students interviewed in this sample was 294 out of 400 points, a grade of B+. The average course grades for the "worksheet" and "traditional" students interviewed were 322 (A-) and 279 (high B) respectively. Both of these are well above the overall course average of 247 points (B-), indicating that our sample of interview students comes from the upper portion of the class.

Interviews typically followed the format of the free-response diagnostic, though with enough freedom to pursue questions other than those on the free-response diagnostic. Selected transcripts can be found within the text and in Appendix D.

First-semester vs. Second-semester

In the following chapters, the first and second-semester physics classes will be referred to as "first-semester" and "second-semester" groups. It should be noted that we are not referring to the same set of students at different stages in their education. Rather, they are similar (but not identical) groups of students taking first- and second-semester courses.

After the initial diagnostics were administered in the Fall 1999 Physics 111 and 221 classes at Iowa State, some of the students in those classes received instruction using the worksheets produced by the author (see Appendix C). Because of this, it is necessary to be careful in analyzing data from the Spring 2000 classes. Students who received this non-traditional instruction by the worksheet method were removed from the Spring sample in those cases. Thus, we can consider the Spring 2000 Physics 112 and 222 classes as post-traditional instruction.

In addition, the population from which students in the second-semester classes come is not identical to the first-semester student population. Part of the student population that takes the first-semester course never takes the second course. This can be due to individual choice, performance in the first semester course, or the lack of a requirement for a second semester of physics for the student. As an example, students majoring in civil engineering are not required to take a second semester of physics.

Worksheets

As the results in Chapter 4 will show, there is significant room for improvement in teaching students about gravity, especially in helping them to achieve conceptual understanding of the subject. In an effort to improve upon traditional instruction, a set of worksheets was produced to address some of the common problems students had exhibited in their responses to the diagnostic questions. These worksheets are written in a format similar to *Tutorials in Introductory Physics*[35] produced by the Physics Education Group at the University of Washington. The worksheets were produced with the intention of making them useful to students in traditional physics and astronomy classes, while not taking up large amounts of class time.

To be this flexible, the worksheets had to meet several requirements. First, introductory material had to be included on what forces are and how to represent them in diagrams. This was necessary since most introductory astronomy students lack formal instruction in physics and only know about the term force as it is loosely used in common language. Second, since the goal was to use the worksheets within a single class period or fraction thereof, the worksheets had to be no more than a few pages in length. Third, the worksheets had to address topics that proved to be major stumbling blocks for students. This was done with the information taken from the diagnostics.

More detail about the diagnostics can be found in Chapter 5, and the worksheets can be viewed in their entirety in Appendix C.

CHAPTER 4. STUDENT CONCEPTS OF GRAVITY

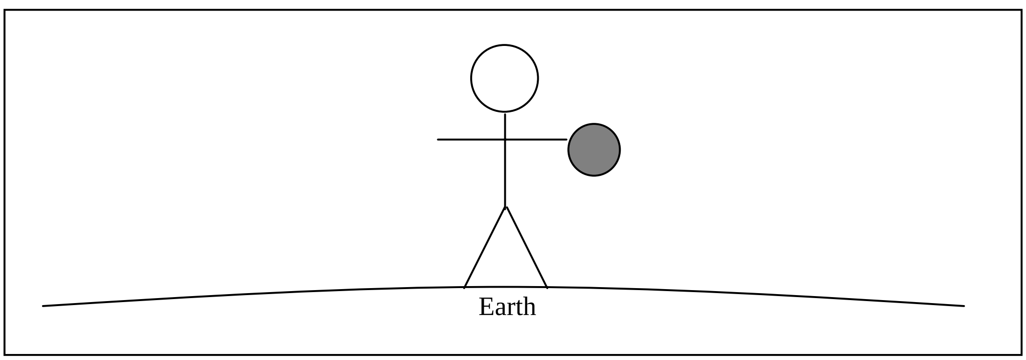
Direction and Superposition of Gravitational Forces

Significance of results

For some questions on the two diagnostics, the differences between first- and second-semester courses (in percentage of students answering correctly) are significant, while for others they are not. We use a two-sample test for binomial proportions[36] to judge if such a difference between first- and second-semester correct-response rates is statistically significant. The proportions we wish to compare are represented by $\hat{p}_1 = X/m$ and $\hat{p}_2 = Y/n$, which are ratios of the number of students answering a question favorably (*X*, *Y*) to the total number of students questioned (*m*,*n*). $\hat{p}_1$ and $\hat{p}_2$ represent the ratios for the first and second-semester classes respectively. When more than one semester of a particular course is quoted in a table, the student responses are combined for that course (before calculating $\hat{p}_1$ and $\hat{p}_2$) if and only if the differences between the responses for those multiple semesters are not *themselves* statistically significant. (If they are significantly different, we omit any calculation of significance of the difference between first- and second-semester courses.) We then use these proportions to calculate the test statistic *z* where $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(1/m + 1/n)}}$, $\hat{p} = \frac{m}{m+n}\hat{p}_1 + \frac{n}{m+n}\hat{p}_2$, and $\hat{q} = 1 - \hat{p}$. For a two-tailed *z* test, the *p*-value is calculated as $p = 2[1 - \Phi(|z|)]$. The *p*-value is quoted in each table where such a comparison is appropriate to make, where $p < 0.005$ is chosen to represent a statistically significant difference in correct answer rates between samples of students. We choose $p < 0.005$ as the criterion for statistical significance due to the large number of comparisons made among different questions. p-values will not be tabulated when one or more of the samples has $N < 30$.

Free response #1 and #10

Discussion of the direction of gravitational forces will be treated briefly before moving to other aspects of gravity. Free-response diagnostic questions #1 and #10 are both questions about the direction of the force of gravity. Question #1 shows a person standing on the Earth while holding a ball in one hand.



In the picture above, a person is standing on the Earth holding a ball in one hand. Draw the direction of the gravitational force exerted ***by*** the Earth ***on*** the ball in the picture above.

Figure 4-1. Free-response diagnostic #1.

The student is asked to draw the direction of the force exerted by the Earth on the ball. Results from #1 do not indicate any appreciable differences between the students in the algebra-based physics classes and the calculus-based physics classes. In both, the first-semester students do very well on this question, with 92% of the class answering correctly.

The most common incorrect response was to draw an arrow pointing upward, as if the ground were reaching up to the ball in order to pull it down. There is no significant change in the percentage of correct answers when considering students in the second semester of their respective courses. Unlike the first-semester students, these students have had traditional lecture instruction on gravity concepts.

Table 4-1: Free response #1, calculus-based results.

	First semester	***Second semester***
	ISU Phys221 Fall 1999 (N = 539)	ISU Phys222 Fall 1999 (N = 189)
Up arrow	6%	7%
Down arrow (correct)	**92%**	**92%**
Other/No response	2%	1%

Table 4-2: Free response #1, algebra-based results.

	First semester	***Second semester***
	ISU Phys111 Fall 1999 (N = 302)	ISU Phys112 Summer 1999 (N = 21)
Up arrow	6%	10%
Down arrow (correct)	**92%**	**90%**
Other/No response	3%	0%

Question #10, which is very similar to an interview question asked of elementary and middle school students in prior researchers' works[11-13], shows a complete round Earth with stick figures standing at different places on the surface, each with a ball in one hand. Students are asked to draw the direction they think the ball will fall when released. Unlike #1, the word "force" is never introduced. For the calculus-based classes, the results for the first- and second-semester students were 96% correct (N = 518) and 100% correct (N = 183) respectively. Results for the algebra-based classes are similar (92% correct first-semester and 100% correct second-semester.) Appropriately, these results are an improvement over Nussbaum's results for high school students, perhaps suggesting that the trend of improvement on this concept continues with advancing age and grade level through college as well.

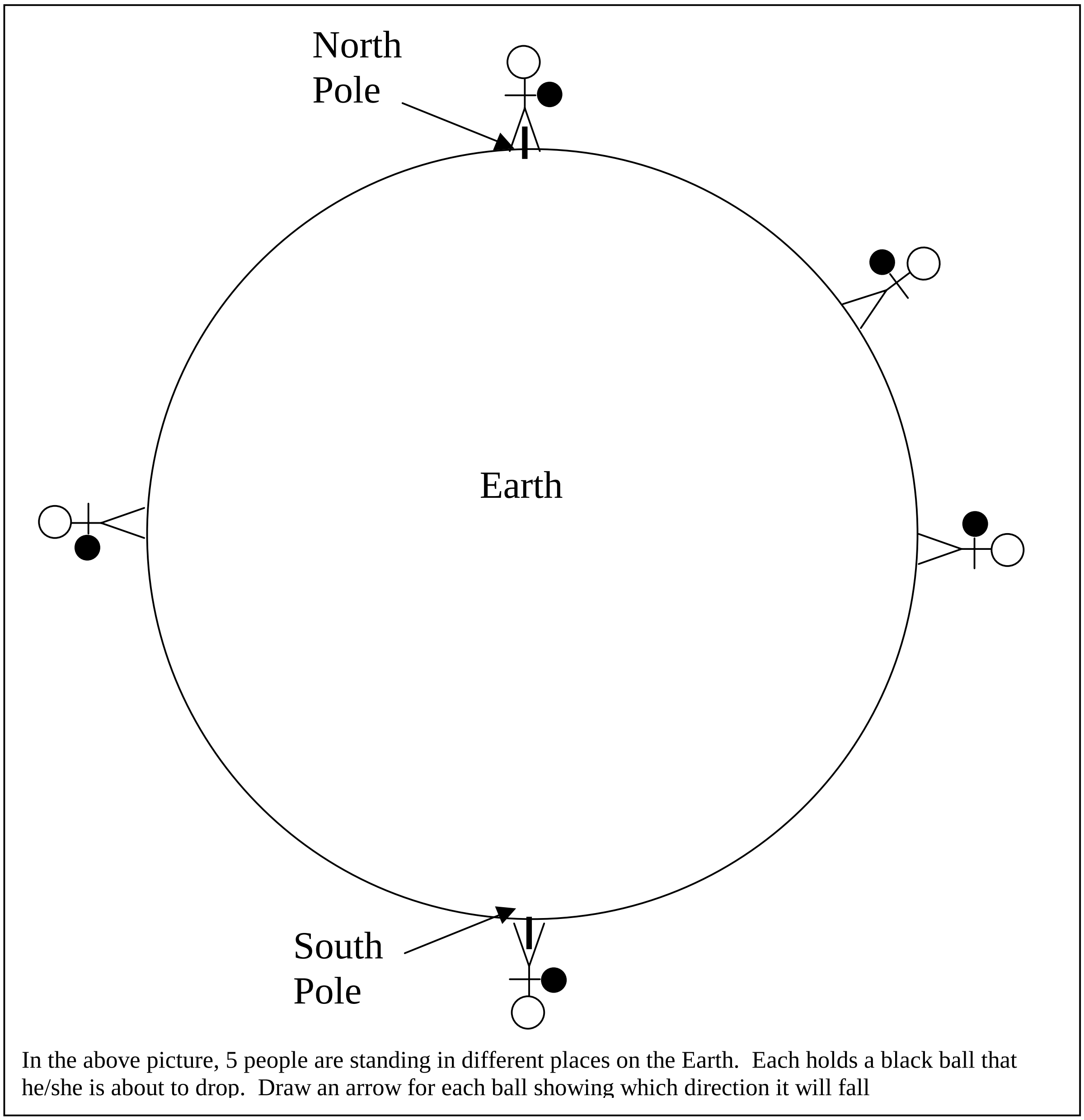
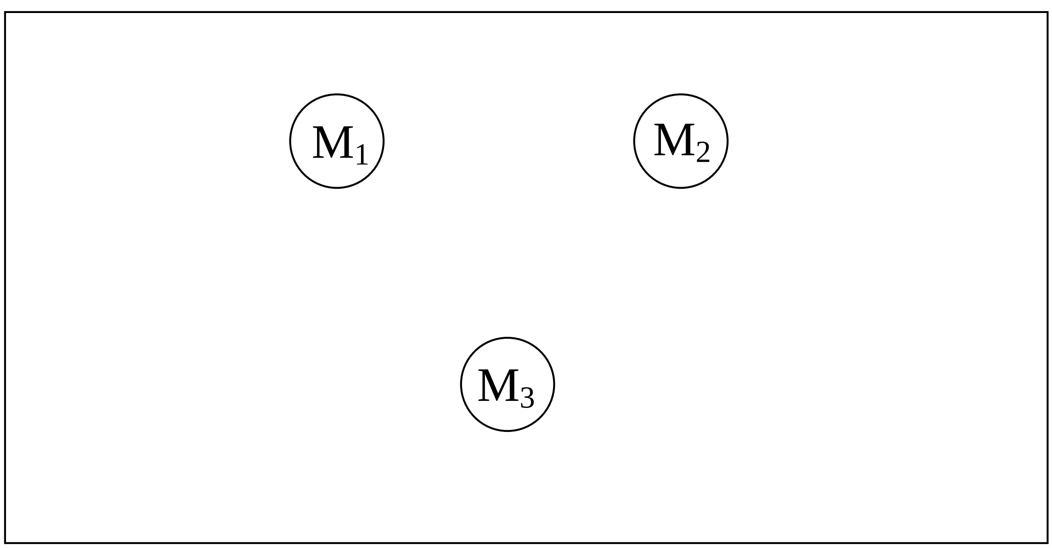


Figure 4-2. Free-response diagnostic #10

Free response #6

Question #6 allows us to look at the students' use of superposition of forces in a gravitational context. The results for the calculus-based class show 51% of first-semester students answering correctly. Second-semester results are better, but only up to 68% correct for the Physics 222 class. The explanations for the correct answers are mixed; approximately half of the responses refer to vector addition, either explicitly or inferred. The other half of the explanations are divided between descriptions of the moon's motion and responses that lack explanations. About 40% of the first-semester students give responses saying that M_3 will not move. The explanations for this answer are varied, and no single explanation is given by a majority of the students. The most common explanation, given by about one-third



In the above diagram, three large moons are arranged so that they make an equilateral triangle. All three are the same size and have the same mass. Moons M_1 and M_2 are fixed in position and ***can not move.*** Moon M_3 is initially at rest, but is free to move.

Will moon M_3 move? [circle one] **YES** **NO**

If yes, draw an arrow to indicate the direction that M_3 will move, and explain the reason for your answer. If no, explain why M_3 does not move.

Figure 4-3. Free-response diagnostic #6.

Table 4-3: Free-response diagnostic #6, calculus-based results.

	First-semester		***Second-semester***	
	ISU Phys221		ISU Phys222	Post-trad.
	Summer 1999	Fall 1999	Fall 1999	interviews
Response	(N = 41)	(N = 546)	(N=189)	(N = 17*)
Yes, moves up (correct)	**56%**	**51%**	**68%**	**94%**
Yes, other	5%	3%	4%	0%
No movement	37%	41%	28%	6%
Don't know	0%	1%	0%	0%
No Response	0%	1%	0%	0%

*#6 was not asked in one of the interviews

of the first-semester students, states that "the forces are equal," and because of that M_3 does not move. This is a somewhat vague statement. It is sometimes clarified as "the forces from M_1 and M_2," "all the forces," or not clarified at all.

For the algebra-based students, the first-semester performance is similar (50% correct). The second-semester performance is noticeably worse (24% correct), but is a sample of only 21 students. The difference between these results is not statistically significant.

Explanations for the algebra-based students' responses are much less varied than the explanations given by the calculus-based students. Of students giving correct responses,

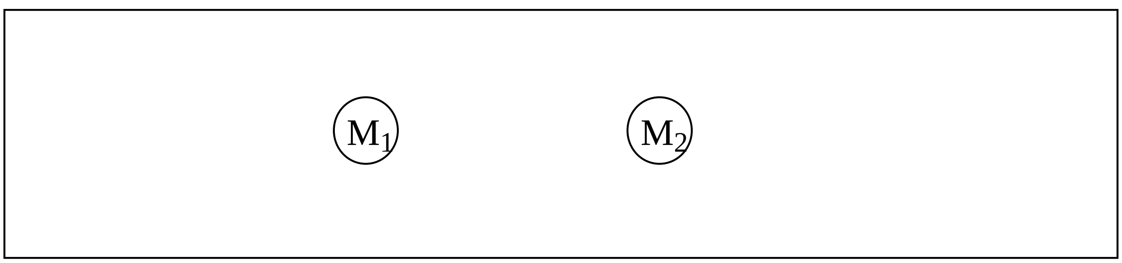
Table 4-4: Free-response diagnostic #6, algebra-based results.

	First-semester	***Second-semester***
	ISU Phys111	ISU Phys112
	Fall 1999	Summer 1999
Response	(N = 303)	(N = 21)
Yes, moves up (correct)	**50%**	**24%**
Yes, other	1%	5%
No movement	46%	71%
Don't know	1%	0%
No Response	3%	0%

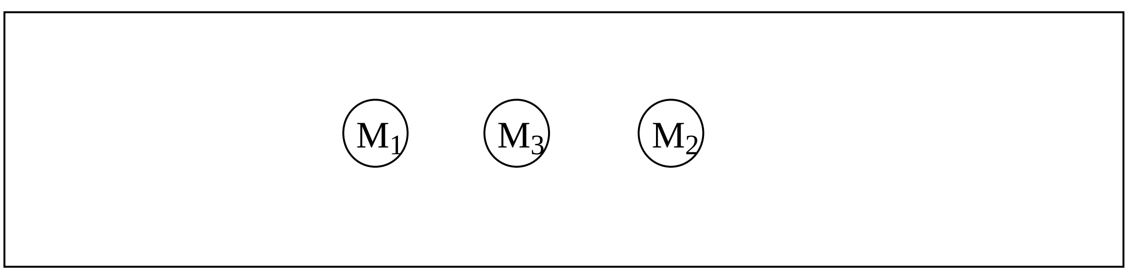
more than two-thirds of the students explain that M_1 and M_2 pull M_3 equally. Ironically, the most common explanation given by more than half the students for the *in*correct answer is that "the forces are equal." This is a reasonably similar phrase to that given as an explanation for the correct answer.

Free-response #7a and multiple choice #10

These two questions are nearly identical, both involving the insertion of a third mass between two masses already present. Students are asked to determine whether the net gravitational force on M_2 changes.



Two large masses M_1 and M_2 are in space as shown above.
A third mass M_3 is now placed in the position shown below.



A) [circle one] The net gravitational force on M_2 is now (**GREATER THAN,** **LESS THAN,** **THE SAME AS**) it was before M_3 was introduced. Explain your answer.

Figure 4-4. Free-response diagnostic #7a.

The calculus-based students have good results on this question, with 72% correct responses for the first-semester students and 86% for the second-semester students. A majority of the explanations given by these students answering correctly, both first and second-semester, indicated that there were two forces present or that "the forces add." The response that two forces are present could lead to incorrect answers if the mass is inserted in another location, as in free-response #7b (addressed later in this section.)

Table 4-5: Free-response diagnostic #7a, calculus-based results.

	First-semester	***Second-semester***	
Response	ISU Phys221 Fall 1999 (N = 546)	ISU Phys222 Fall 1999 (N = 189)	Post-trad. interviews (N=17*)
Greater force (correct)	**72%**	**86%**	**82%**
Same Force	17%	8%	6%
Less Force	8%	4%	12%
Other/No Response	3%	1%	0%

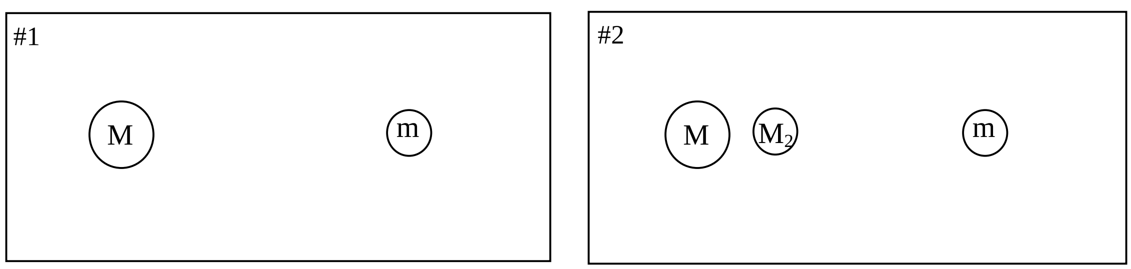
*#7a was not asked in one of the interviews

Table 4-6: Free-response diagnostic #7a, algebra-based results.

	First-semester	***Second-semester***
Response	ISU Phys111 Fall 1999 (N = 302)	ISU Phys112 Summer 1999 (N = 21)
Greater force (correct)	**60%**	**Correct: 81%**
Same Force	28%	Incorrect: 14%
Less Force	8%	
Other/No Response	4%	No response: 5%

The algebra-based students perform similarly to the calculus-based students on #7a, though the percentage of correct responses for the first-semester students is smaller (60%). Again, more than half of the students giving correct responses explain that there are two forces acting on M_2 which add. A larger number of students give an incorrect response that the force on M_2 is the same as before the introduction of M_3. The main explanation given by these students is that "since the distances between all the masses are the same, the force will be the same." Though this explanation is given by a just a few of the calculus-based students, it is the most common explanation given by the algebra-based students. A little more than one-third of the algebra-based students respond this way, and the remainder of the explanations are varied or not given.

In diagram #1, a large mass "M" is near to mass "m." In diagram #2, a smaller mass "M_2" has moved between the other two masses. What will happen to the magnitude of the *net gravitational force* acting on mass "m"?



A. It will increase, due to the force of the additional mass M_2.
B. It will stay exactly the same as it was in diagram #1.
C. If will decrease, because the mass M_2 shields some of the force originally coming from mass M.
D. It is not possible to say whether it will increase, decrease, or remain the same, with the given information.

Figure 4-5. Multiple choice diagnostic #10.

Multiple-choice #10 serves as a check on free-response #7a. It only differs by the placement of the middle mass, which is no longer directly in the middle of the two masses originally present.

73% of the second-semester calculus-based students answer #10 correctly. This is a bit less than the percentage of correct responses for the similar free-response #7a (86% correct). The main difference between the two questions is that multiple choice #10 attaches

Table 4-7: Multiple choice diagnostic #10, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A (correct)	**73%**
B	12%
C	11%
D	4%
No response	0%

Table 4-8 Multiple choice diagnostic #10, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
Response	Spring 2000 (N = 289)	Fall 1998 (N = 79)	Fall 1999 (N = 96)	Spring 2000 (N = 119)
A (correct)	**40%**	**63%**	**64%**	**69%**
B	15%	10%	6%	7%
C	34%	16%	26%	18%
D	10%	10%	4%	6%
No Response	1%	0%	0%	0%

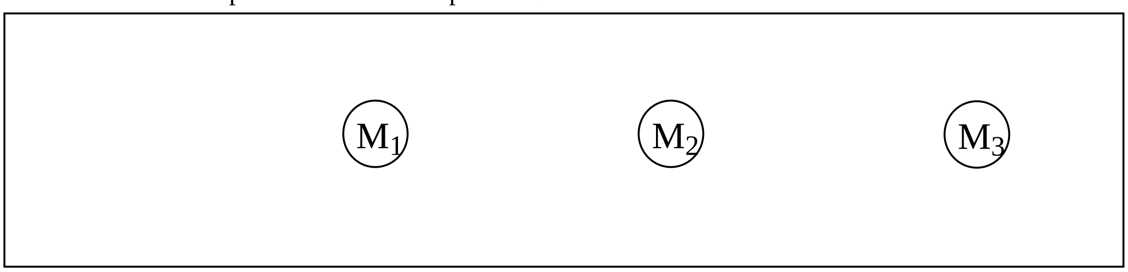
explanations to each of its choices, while that is not done in free-response #7a. Those explanations could possibly affect the choice a student selects.

The algebra-based students also have a lower percentage of correct responses when compared to free-response #7a. The most common incorrect response is C, which states that the force is decreased because the intervening mass shields the force from mass M. For the earlier #7a, shielding was not commonly given as an explanation by the algebra-based students. The best way to determine if students consider shielding to be a valid explanation is to separate the shielding part of the question from the part asking about the net gravitational force.

Free-response #7b

#7b is slightly different than #7a, with that difference being the placement of M_3. This mass is now opposite M_2 and is the same distance away as M_1. The percentage of correct responses is down from #7a for both calculus-based and algebra-based students. For the calculus-based students, about half of those answering correctly say that the forces from M_1 and M_3 cancel one another. However, there is also a significant fraction of the students (about one-fourth) that explain that the masses are now farther apart than they were in 7a, so that the force is less. This suggests that students might be comparing the diagram in 7b with the wrong diagram in 7a.

Now the mass M3 is placed in a different position:



B) [circle one] The net gravitational force on M_2 is now (**GREATER THAN,** **LESS THAN,** **THE SAME AS**) it was before M_3 was introduced. Explain your answer.

Figure 4-6. Free-response diagnostic #7b.

Table 4-9: Free-response diagnostic #7b, calculus-based results.

	First-semester	***Second-semester***	
Response	ISU Phys221 Fall 1999 (N = 546)	ISU Phys222 Fall 1999 (N = 189)	Post-trad. interviews (N=16*)
Greater force	24%	16%	31%
Same Force	23%	11%	6%
Less Force (correct)	**49%**	**72%**	**63%**
Other/No Response	3%	1%	0%

*#7b was not asked in two of the interviews

Table 4-10: Free-response diagnostic #7b, algebra-based results.

Response	***First-semester*** ISU Phys111 Fall 1999 (N = 302)	***Second-semester*** ISU Phys112 Summer 1999 (N = 21)
Greater force	32%	Incorrect: 38%
Same Force	28%	
Less Force (correct)	**34%**	**Correct: 57%**
Other/No Response	6%	No response: 5%

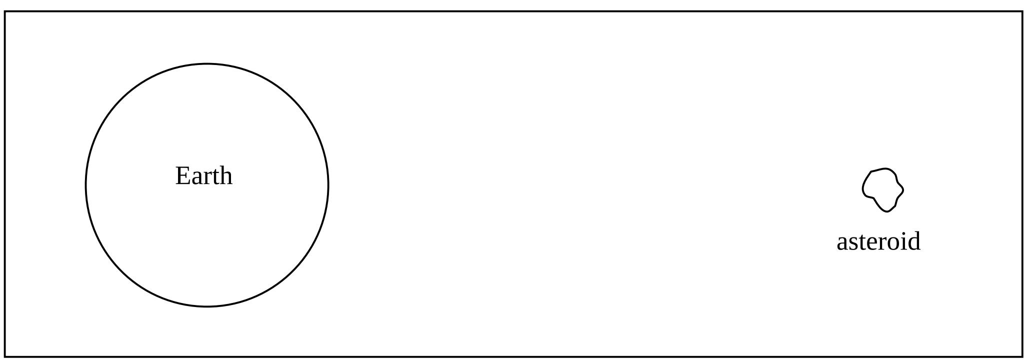
There are two important explanations given for the incorrect responses. First, for students who answered that the force acting on M_2 would be greater, the dominant explanation given by more than half of the students is that there are two forces acting on M_2. This is an explanation that leads to a correct answer in #7a, but an incorrect answer in 7b. For students who answered that the force on M_2 would be the same as before, the most common explanation (given by roughly one-third of those students) is, "the masses are at equal distances from one another, so the force is the same." It is possible that some of the students are comparing the force of M_1 acting on M_2 with the force of M_3 acting on M_2. The question would benefit by changing the wording to leave no doubt about which forces the students are comparing.

Newton's Third Law in the Context of Gravity

The question of how students' concepts of gravity align with Newton's third law is addressed by several questions on both diagnostics. Questions on the diagnostic exams most directly relating to Newton's third law in the context of gravity are #1 and #8 on the multiple-choice diagnostic and #2 and #4 on the free-response diagnostic. The questions are similar in that only two objects are present and no other objects are nearby. Each question asks the student to determine the forces acting on the two objects.

Free-response #2

Free-response #2, shown in Figure 4-7, deals with the case of the Earth and an asteroid isolated in space. Students are asked to select one of four multiple-choice answers, but then explain the reason for their selection. For the calculus-based classes (see Table 4-11), the most common error made by the students before instruction is to claim that the asteroid exerts a smaller force than does the Earth. Approximately three-fourths of the first-semester calculus-based students answer this way. In addition, more than three-fourths of the students responding



Refer to the picture above.

The magnitude of the force exerted **by** the asteroid **on** the Earth is [circle one]:
a) *larger than* the magnitude of the force exerted **by** the earth **on** the asteroid.
b) *the same as* the magnitude of the force exerted **by** the earth **on** the asteroid.
c) *smaller than* the magnitude of the force exerted **by** the earth **on** the asteroid.
d) *zero*. (the asteroid exerts no force on the Earth)

Explain the reasoning for your choice.

Figure 4-7. Free response diagnostic #2.

Table 4-11: Free-response diagnostic #2, calculus-based results.

	First-semester			***Second-semester***	
	ISU Phys221			ISU Phys222	Post-trad.
	Summer 1999	Fall 1999	Spring 2000	Fall 1999	interviews
Response	(N = 41)	(N = 547)	(N = 324)	(N = 414)	(N=18)
A	2%	3%	2%	1%	0%
B (correct)	**12%**	**15%**	**19%**	**38%**	**61%**
C	78%	74%	73%	59%	28%
D	7%	7%	5%	1%	6%
Other/No answer	0%	1%	0%	1%	6%

with this answer justify their claim by saying that since the Earth has more mass, it should exert more force. This holds true for second-semester students as well, for approximately two-thirds of the students retain this view after covering the material. Even when there is cause to think otherwise, students hold to this answer. This is demonstrated in the following second-semester interview transcript.

Student (S): *Because the Earth weighs more, it attracts the asteroid more, but that doesn't justify it. I'm not sure. I just know that because it weighs more it has a larger force on the asteroid.*

Interviewer (I): *By saying it doesn't justify it what do you mean?*

S: *Well, yeah. It doesn't have an equation to back it up. And I can't remember how to manipulate the equation so it says that.*

The students in the post-traditional instruction interviews do perform better on this question than the rest of the second-semester students. Though it is difficult to identify a specific reason for the difference, it is worth noting that the misconception (choice C) is still present in the sample (59%), even after instruction, and even with some of the top students in the interview sample.

Also, a small yet significant fraction of the students select answer D, that the asteroid exerts no force on the Earth. The reasoning for this response varies. Typical examples of reasoning are that there is no force in space or that the asteroid is too small to "have its own gravity."

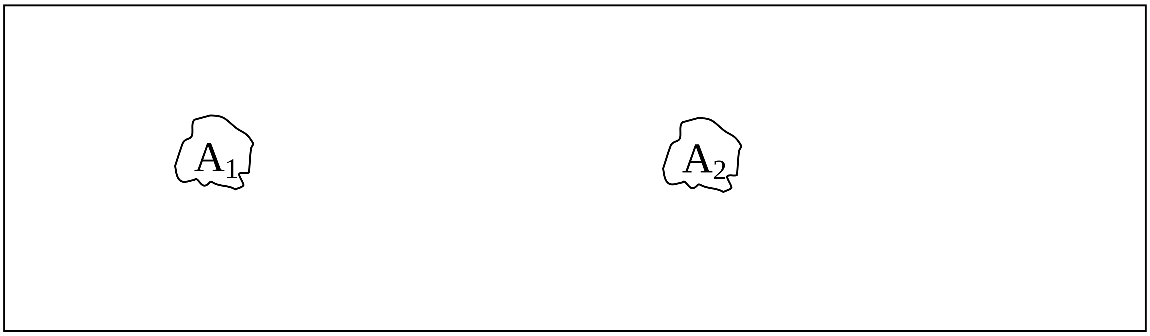
Results for the algebra-based students in Table 4-12 are similar to the results for the calculus-based students. Responses are very heavily weighted toward the answer that the Earth and the asteroid do not exert the same force on one another. Again, the overwhelming explanation (more than three-fourths of the responses) for the difference in forces is the fact that the Earth has much more mass than the asteroid. In addition, it is notable that even after instruction, the situation is not greatly improved. Regardless of the level of the course, the inequality of force between two dissimilar objects seems to be one of the most widespread misconceptions among physics students.

Table 4-12: Free-response diagnostic #2, algebra-based results.

	First-semester		***Second-semester***
	ISU Phys111		ISU Phys112
Response	Summer 1999 (N = 48)	Fall 1999 (N = 303)	Summer 1999 (N = 21)
A	2%	4%	5%
B (correct)	**13%**	**13%**	**24%**
C	63%	69%	67%
D	23%	14%	5%
Other/No answer	0%	1%	0%

Free-response #4

In free-response #4 (Figure 4-8), two asteroids are present in space, isolated from all other bodies so that the only forces they feel are those of gravitational attraction due to one another. This is again a case of Newton's third law in action, but now in a situation that does not involve the Earth. Consistent with the results from free-response #2, a large majority of



The picture shown is a region of space showing two large, massive asteroids labeled A1 and A2. Both asteroids are free to move. **Assume nothing else is present**. Although the two asteroids are the same ***size***, asteroid A1 is ***three times as massive*** as asteroid A2. For each asteroid, draw one or more arrows on the asteroid to indicate the directions of any pushes or pulls which that asteroid experiences.

If two pushes or pulls are the ***same strength***, draw the arrows representing them ***the same length***. If one push is ***stronger*** than another one is, draw a ***longer*** arrow to indicate the stronger push/pull. Draw a ***shorter*** arrow to indicate a ***weaker*** push/pull.

Explain the reason for your answer.

Figure 4-8. Free-response diagnostic #4.

the students, both first and second-semester, sketch the gravitational force acting on the asteroids as not being equal. Nearly all the students giving explanations quote the mass difference as the reason for that answer. In the Summer 1999 Physics 221 class for example, two-thirds of the students responding incorrectly specifically stated that since A_1 had a greater mass, it exerted a greater force. The next most common response was no explanation, given by one-fourth of the students.

Table 4-13: Free response diagnostic #4, calculus-based results.

	First-semester		***Second-semester***
	ISU Phys221		ISU Phys222
	Summer 1999 (N = 41)	Fall 1999 (N = 546)	Fall 1999 (N = 189)
Forces equal (correct)	**10%**	**7%**	**23%**
Forces unequal	85%	80%	69%
No forces present	0%	1%	2%
Other/Don't Know	2%	9%	5%
No Response	2%	3%	1%

Results for the algebra-based students (Table 4-14) are virtually the same as those for the calculus-based class, with overall performance on the question being slightly poorer. For the students answering incorrectly, both first and second-semester, students give the difference in mass as the reason that the forces should be unequal.

Table 4-14: Free-response diagnostic #4, algebra-based results.

	First-semester	***Second-semester***
Response	ISU Phys111 Fall 1999 (N = 303)	ISU Phys112 Summer 1999 (N = 21)
Forces equal (correct)	**4%**	**0%**
Forces unequal	79%	90%
No forces present	2%	0%
Other/Don't Know	8%	10%
No Response	6%	0%

Multiple-choice #1 and #8

In addition to the free-response questions, two questions on the multiple-choice diagnostic (#1 and #8) are relevant to this topic. #1 asks how the force exerted by the Sun on the Earth compares to the force of the Earth pulling on the Sun, noting that the Sun is much more massive than the Earth. To answer correctly, the student must recognize the Earth and Sun as an action-reaction pair and that the force that each exerts on the other is the same.

Multiple-choice diagnostic #1:

The mass of the Sun is about 3×10^5 times the mass of the Earth. How does the magnitude of the gravitational force exerted by the Sun on the Earth compare with the magnitude of the gravitational force exerted by the Earth on the Sun? The force exerted by the Sun on the Earth is:

A. about 9×10^{10} times larger
B. about 3×10^5 times larger
C. exactly the same
D. about 3×10^5 times smaller
E. about 9×10^{10} times smaller

For the calculus-based students, we have only second-semester data. The second-semester calculus-based students do much better on this question than they do on free-response #2 and #4 (see Table 4-15). The most common error, consistent with the free-response diagnostic results, is responding that the larger object is the one exerting the larger force.

There is no readily apparent reason for the difference in performance, so it is open to speculation. Students taking Physics 222 who were instructed using the worksheet method while taking Physics 221 during the Fall 1999 semester were removed from the sample, so they are not the cause of the difference. One possibility is that the representations of the questions are sufficiently different to cause this difference in percentage of correct responses. Another speculation is that the students may remember a similar question from the previous semester. During the Fall 1999 semester, a final exam question on a similar topic was asked of all of the Physics 221 students (see Chapter 5). The speculation is that some of the students determined the correct answer to that question after seeing it on the final, and applied that knowledge to the diagnostic question.

Table 4-15: Multiple-choice diagnostic #1, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A	6%
B	32%
C (correct)	**62%**
D	1%
E	0%
No response	0%

Table 4-16: Multiple choice diagnostic #1, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
	Spring 2000	Fall 1998	Fall 1999	Spring 2000
Response	(N = 289)	(N = 79)	(N = 96)	(N = 119)
A	11%	13%	10%	11%
B	64%	68%	73%	45%
C (correct)	**17%**	**14%**	**10%**	**41%**
D	7%	5%	6%	3%
E	0%	0%	0%	0%
No Response	1%	0%	0%	0%

The algebra-based students have more pronounced difficulties with this problem, answering only 17% correct first-semester and 10% to 41% correct second-semester. Also, there is a noticeable difference in performance between the Spring 2000 Physics 112 class and the fall Physics 112 classes. Additional sampling of students is needed to isolate the cause of this variation.

Question #8 on the multiple-choice diagnostic (Figure 4-9) asks a question similar to

Which of these diagrams most closely represents the gravitational forces that the Earth and moon exert ***on each other***? (Note: the mass of the Earth is about 80 times larger than that of the moon)

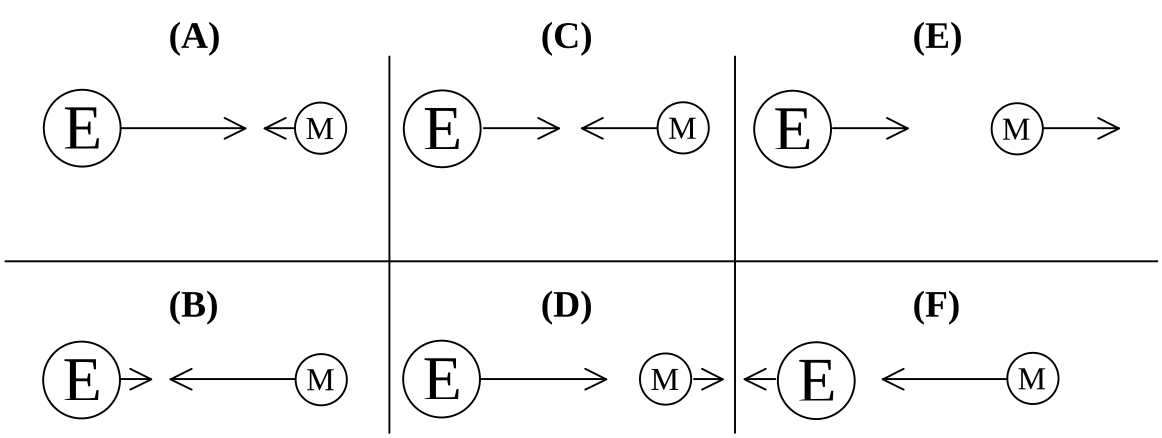


Figure 4-9. Multiple-choice diagnostic #8.

Table 4-17: Multiple-choice diagnostic #8, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A	26%
B	32%
C (correct)	**38%**
D	3%
E	0%
F	1%
No response	0%

#1. However, this time students have six diagrams to choose from rather than text-based choices. Students again must recognize that Newton's third law is directly applicable here and select the choice in which the arrows are the same length and pointing toward one another.

The second-semester calculus-based results are surprisingly different than their results for #1. While 62% of the students answer #1 correctly, only 38% of the students answer #8 correctly! This suggests that a problem’s representation can significantly affect student responses about that physical principle. We cannot assume that a student’s understanding of a concept is robust based on answers to a single representation of a problem. The student performance on these two questions is consistent with results of a study by Meltzer[23]. Meltzer classifies #1 as a “Verbal” question and #8 as a “Diagrammatic” question. The major difference between the student performance on the two questions is that choice “A” on question #8 (larger mass exerts smaller force) is chosen more frequently than the corresponding responses (D and E) on question #1.

Results for the algebra-based students (Table 4-18) follow the same trend. Fewer of the algebra-based students, both first- and second-semester, answer #8 correctly. The variance in the second-semester data is present again, as it was for question #1.

For the second-semester students in the calculus- and algebra-based courses, the incorrect responses given are for the most part split evenly between answers A and B. These two choices show both force arrows pointing inward, but the longer arrow is attached to the Earth in A and to the moon in B. This split of responses suggests a possibility that students' representations of forces do not follow a single convention, even after instruction over such material.

Table 4-18: Multiple choice diagnostic #8, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
	Spring 2000	Fall 1998	Fall 1999	Spring 2000
Response	(N = 289)	(N = 79)	(N = 96)	(N = 119)
A	59%	38%	47%	32%
B	24%	53%	45%	29%
C (correct)	**8%**	**6%**	**6%**	**36%**
D	2%	0%	0%	1%
E	2%	1%	0%	1%
F	4%	0%	2%	1%
No Response	0%	1%	0%	0%

Newton's Law of Gravitation

When students in physics classes study the subject of gravity, the instruction normally includes Newton's law of gravitation, $F=GM_1M_2/r^2$. When testing students' knowledge of this force law, the least complex way to do so is to change a single variable and ask, "What happens to the force?" The instructor's hope is that the student will recognize that it is appropriate to use this law in solving the problem, and then apply that law correctly. Four questions on the diagnostics (multiple choice #2, #3, and #9 and free-response #3) directly address the law of gravitation by changing one or more of these variables at once.

Multiple choice #3

Question #3 on the multiple choice diagnostic asks how changing the mass of the Sun affects the force it exerts on the Earth. To answer correctly, students need to understand the relationship between the object's own mass and the force it exerts on another object. This is a linear relationship, so when an object itself doubles in mass, the force it exerts on another object also doubles.

Multiple-choice diagnostic, #3:

Suppose the distance between the Earth and the Sun stay the same, but somehow the mass *of the Sun were doubled. What would happen to the magnitude of the gravitational force exerted by the Sun on the Earth?*

A. It would be four times larger
B. It would be twice as large
C. It would be exactly the same
D. It would be half as large
E. It would be one-fourth as large

In Table 4-19, we see that the correct answer (B) is chosen by 86% of the Physics 222 students. Although there are no data for first-semester students, the data presented seem to indicate that traditional instruction seems to be sufficiently effective in addressing this point.

The algebra-based students follow the same trend, with 71% of the first-semester students answering correctly and around 80% of the second-semester students answering correctly. In both the calculus- and algebra-based classes, the most common incorrect response is A, that the force is quadrupled when the mass is doubled. This would indicate an incorrect application of a square dependence.

Table 4-19: Multiple-choice diagnostic #3, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A	9%
B (correct)	**86%**
C	4%
D	0%
E	1%
No response	0%

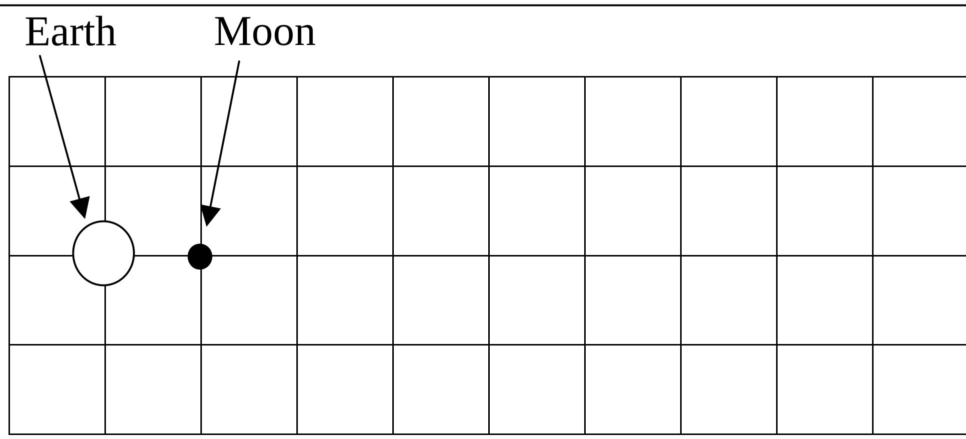
Table 4-20: Multiple-choice diagnostic #3, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
Response	Spring 2000 (N = 289)	Fall 1998 (N = 79)	Fall 1999 (N = 96)	Spring 2000 (N = 119)
A	19%	15%	10%	12%
B (correct)	**71%**	**76%**	**83%**	**81%**
C	8%	8%	5%	4%
D	1%	1%	1%	3%
E	1%	0%	0%	1%
No Response	0%	0%	0%	0%

Free-response #3 and multiple choice #2

These questions address how changing the distance between two bodies affects the force acting on them. In multiple-choice #2, the distance between the Sun and Earth is doubled, and the student is asked for the force. Conversely, in free-response #3 the student is given the force (one-fourth as much) and asked to provide the distance. In both cases, the student must understand the inverse-square relationship between the separation distance and the gravitational force experienced.

For free-response #3, 20 to 22% of the first-semester calculus-based students answer correctly. More than half of the students giving these correct answers refer to an inverse square law or directly to Newton's law of gravitation. In contrast, 40% of the class answers that moving the moon 4 times as far away gives a force that is one-fourth as strong. About two-thirds of this 40% give explanations that are linear in nature, either expressly using words such as "proportional," "linear," or by simply saying that four times the distance gives one-fourth the force. The remainder of the responses are divided between "no explanation" and "arithmetic sequence" responses. The arithmetic sequence response is one in which the student effectively labels the grid squares in an arithmetic sequence: at 1 unit from the Sun (where the moon is at first), the force is F, 2 units is 3/4 F, 3 units is 1/2 F, and 4 units is 1/4 F. It is also worth noting that many of the students placing the Earth three times or five times as far away from the Sun give reasoning that is consistent with placing the Earth two or four times as far away from the Sun. This indicates confusion between what it means to move an object two (or four) more units away and moving an object twice (or four times) as far away from the other object. This problem persists in the second-semester data, as the second-semester calculus-based students, while performing better than the first-semester students, still have similar problems.



The Moon experiences a gravitational force due to the Earth. Let's call the magnitude of this force $F_{\text{Earth on Moon}}$.

If you could pick up the moon and move it to another position, where would you put it to reduce the magnitude of $F_{\text{Earth on Moon}}$ to ¼ of its original value? Indicate the new position of the moon on the picture above and explain why you chose that position.

Figure 4-10. Free-response diagnostic #3.

Table 4-21: Free-response diagnostic #3, calculus-based results.

	First-semester		***Second-semester***	
	ISU Phys221		ISU Phys 222	Post-trad.
	Sum. 1999	Fall 1999	Fall 1999	interviews
Response	(N = 41)	(N = 546)	(N=189)	(N=18)
2x away (correct)	**22%**	**20%**	**44%**	**61%**
3x	10%	13%	11%	0%
4x	41%	39%	28%	17%
5x	7%	7%	8%	11%
Other (farther away)	20%	15%	7%	11%
Other (closer)	0%	2%	1%	0%
Don't know	0%	1%	0%	0%
No Response	0%	2%	1%	0%

The algebra-based results in Table 4-22 show that only 10% of the first-semester students answer correctly, and there is not much improvement on this question after instruction. When compared to the calculus-based results, there is a larger percentage of "3x" and "5x" answers. Again, many of these answers indicate reasoning for 2x and 4x

Table 4-22: Free-response diagnostic #3, algebra-based results.

Response	***First-semester*** ISU Phys 111 Fall 1999 (N = 302)	***Second-semester*** ISU Phys 112 Summer 1999 (N = 21)
2x away (correct)	**10%**	**19%**
3x	19%	24%
4x	37%	24%
5x	13%	10%
Other (farther away)	18%	19%
Other (closer)	2%	0%
Don't know	1%	0%
No Response	1%	0%

answers, suggesting that the confusion mentioned for the calculus-based students is a bit more widespread for the algebra-based students.

Multiple-choice #2 serves as a check against free-response #3. In this question, students are given the variable change (that the separation distance is doubled) and asked to determine what happens to the force exerted by the Sun on the Earth. Again, this requires the application of the force-distance inverse square relationship in Newton's law of gravitation.

Multiple-choice #2:

The distance between the Earth and the Sun is about 9.3 x 10^7 miles. If this distance were somehow doubled, what would happen to the magnitude of the gravitational force exerted by the Sun on the Earth?
A. It would be four times larger
B. It would be twice as large
C. It would be exactly the same
D. It would be half as large
E. It would be one-fourth as large

Results from both the calculus-based and algebra-based students show varying percentages of students answering correctly, ranging from 21% (first-semester algebra-based students) to 65% (second-semester calculus-based students). It is worth noting that in each of the classes, 80-95% of the students gave D or E as an answer, the two choices that said the force would decrease. This suggests that while students in both algebra- and calculus-based classes have some trouble with the application of the inverse-square law, most of them do understand that as the objects are moved farther apart, the force exerted by one object on another decreases.

Table 4-23: Multiple-choice diagnostic #2, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A	2%
B	1%
C	1%
D	30%
E (correct)	**65%**
No response	0%

Table 4-24: Multiple-choice diagnostic #2, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
Response	Spring 2000 (N = 289)	Fall 1998 (N = 79)	Fall 1999 (N = 96)	Spring 2000 (N = 119)
A	2%	0%	2%	4%
B	8%	5%	4%	8%
C	6%	5%	3%	2%
D	63%	42%	54%	32%
E (correct)	**21%**	**48%**	**36%**	**55%**
No Response	0%	1%	0%	0%

Multiple choice #9

Question #9 is a more complex question than the others in this section. Here, both mass and distance are varied at the same time. In order to give a correct response, a student needs to understand both how the separation distance and the mass of the objects involved affect the force experienced by one of the masses. In addition, they must be able to apply them both at the same time. The gravitational force drops off as the inverse square of the distance, while it increases linearly with an object's mass. Thus, the force is greatest in diagram I.

We would expect that the percentage of correct answers to multiple choice #9 would be less than those for the other multiple choice questions in this section, since the question relies on proper understanding and application of how both the mass and the separation distance affect the force. This is what we find; results for each of the classes is a little lower than the percentage of correct answers for multiple-choice #3 (and much less than multiple-choice #2, on which the students performed well). The most common incorrect response was F, that the force was the same in all three cases.

What can you say about the magnitude of the gravitational force on the 2 kg sphere in these three situations? The magnitude of the gravitational force on the 2 kg sphere is:

A. largest in I
B. largest in II
C. largest in III
D. equal in I and II, but larger than in III
E. equal in II and III, but larger than in I
F. equal in all three cases

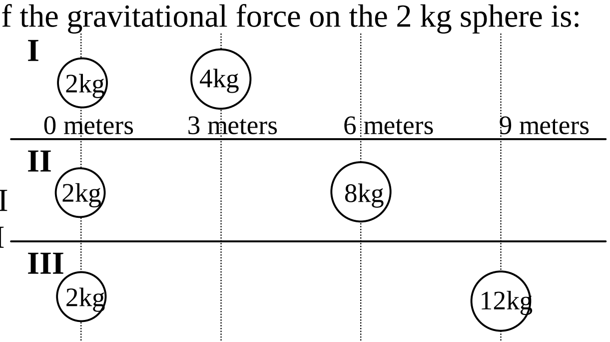


Figure 4-11. Multiple-choice diagnostic #9.

Table 4-25: Multiple-choice diagnostic #9, calculus-based results.

Response	***Second-semester*** ISU Phys222 Spring 2000 (N = 240)
A (correct)	**50%**
B	0%
C	7%
D	6%
E	1%
F	35%
No response	0%

Table 4-26: Multiple choice diagnostic #9, algebra-based results.

	First-semester	***Second-semester***		
	ISU Phys111	ISU Phys112		
Response	Spring 2000 (N = 289)	Fall 1998 (N = 79)	Fall 1999 (N = 96)	Spring 2000 (N = 119)
A (correct)	**28%**	**35%**	**24%**	**46%**
B	4%	0%	3%	1%
C	14%	18%	6%	11%
D	11%	4%	8%	5%
E	3%	3%	2%	3%
F	39%	41%	55%	34%
No Response	1%	0%	1%	0%

Universality of Gravity

Students who take physics classes often recognize a given homework or exam problem as a "physics" problem, so they know they are supposed to use "what they learned in class" to solve the problem. However, it is also possible to ask questions involving physics that do not necessarily trigger that response. These could be referred to as "common sense" questions. In discussing gravity specifically, one can ask these sorts of common sense questions about the nature of events in places other than the Earth's surface. Such questions can serve as an interesting test of how universal the concept of gravity is in the students' minds. In other words, they know that gravity is applicable to events on the surface of the Earth, but what about places like the moon and the space shuttle?

Two multi-part questions on the free-response diagnostic (#8a and #9) deal with this idea directly.

Free response #8a

Imagine that an astronaut is standing on the surface of the moon holding a pen in one hand.

A) If that astronaut lets go of the pen, what happens to the pen? Why?

Table 4-27: Free response diagnostic #8a, calculus-based results.

	First-semester			***Second-semester***
	Summer 1999 (N = 40)	ISU Phys221 Fall 1999 (N = 534)	Spring 2000 (N = 302)	ISUPhys222 Fall 1999 (N = 414)
Drops (correct)	**73%**	**66%**	**68%**	**75%**
Floats	10%	19%	14%	12%
Floats Away	15%	12%	11%	11%
Other/No response	2%	3%	7%	1%

The results of #8a are good; between two-thirds and three-fourths of the calculus-based class answers this problem correctly. However, these calculus-based physics students are the type of people we might expect to answer this question nearly 100% correctly. When asked about this question in an interview, one of the students responded this way:

I: *What's going to happen to that pen?*

S: *I think that'd it just stay there.*

I: *What are you thinking is the reason for the pen just staying there?*

S: *Well I know - that there's no gravitation from force that the moon has that would act on that pen.*

I: *Okay.*

S: *Because it's not it's not like the Earth. So it wouldn't drop towards the moon. Because the moon…well it has a gravitational force on the Earth but it doesn't have a gravitational force on … itself. [makes a gripping gesture with fist, like squeezing a ball] So I think there'd be no reason for it fall, and I don't think it would. I don't think it would just fly off because he's just letting it go. He's not throwing it or anything like that.*

The student's comments nicely summarize the most common explanations students have when claiming the pen will float. About two-thirds of the students saying that the pen will float attribute it to there being no gravity on the moon, or that if there is gravity, there's not enough to pull the pen down.

The same questions were asked of students in the algebra-based physics classes as well. While the percentage of correct answers is less than that of the calculus-based students, the justification for the floating and floating away responses is identical. Again, well over two-thirds of the students cite "not enough gravity" or "no gravity" on the moon for their reasoning.

Table 4-28: Free response diagnostic #8a, algebra-based results.

	First-semester		***Second-semester***
	ISU Phys111		ISU Phys112
	Summer 1999 (N = 48)	Fall 1999 (N = 303)	Summer 1999 (N = 21)
Drops/Falls (correct)	**40%**	**42%**	**38%**
Floats	31%	34%	38%
Floats Away	29%	22%	19%
Other/No response	0%	2%	5%

Free-response #9

Question #9 was also asked, this time looking at the force of gravity as it applies to air and space vehicles and their passengers. The wording of "weightlessness" was specifically avoided in the question because of the possible variance in the students' definitions of weightlessness. Thus, if weightlessness were to be considered in the issue, it would be purely by the choice of the student. Students had little trouble with #9a (see Table 4-29), regarding the existence of the Earth's pull on a plane and on a passenger inside the plane. More interesting was #9b, regarding the pull of the Earth on the space shuttle and an astronaut:

Free response #9b:

Now imagine that you are in the Space Shuttle orbiting the Earth.
i) [circle one] Does the Earth exert a gravitational force on the Shuttle? ***YES NO***
ii) [circle one] Does the Earth exert a gravitational force on you? ***YES NO***
Explain why or why not for both i) and ii).

Table 4-29: Free response diagnostic #9b, calculus-based results.

	First-semester		***Second-semester***
	ISU Phys221		ISUPhys222
Response	Summer 1999 (N = 41)	Fall 1999 (N = 546)	Fall 1999 (N = 189)
i) - % giving correct responses	90%	85%	87%
ii) - % giving correct responses	56%	65%	73%

Students had very few difficulties with part i). Of the few students who answer that there is no gravitational force acting on the space shuttle, the most common reasoning is that there is no gravity in space. More difficulties are found as students answer part ii), with approximately one-quarter to one-third of the students answering incorrectly.

Similar results for the algebra-based class are found for free-response #9b. The students are especially troubled by part ii), with only approximately half of the students answering that the Earth does exert a force on the astronaut in the shuttle. A majority of the students answering incorrectly are swayed by the apparent weightlessness of the astronaut. Thus, they generate one of a number of justifications for there to be no force on the astronaut, such as being too far away from the Earth or that there is no gravity in space.

Table 4-30. Free response diagnostic #9b, algebra-based results.

	First-semester		***Second-semester***
	ISU Phys111		ISUPhys112
	Summer 1999	Fall 1999	Summer 1999
Response	(N = 48)	(N = 301)	(N = 21)
i) - % giving correct responses	77%	75%	71%
ii) - % giving correct responses	44%	49%	41%

CHAPTER 5. INTERVENTION AND EVALUATION

Worksheet Instruction

Standard instruction in the first-semester physics courses at Iowa State consists of lecture, laboratory, and recitation. Recitation periods are typically run in a problem-solving mode, where the instructor (usually a graduate teaching assistant) works problems on the board for students. We attempted a small modification in the recitation instruction of both the algebra- and calculus-based first-semester physics classes during the Fall 1999 semester. Our goal was to see if it was possible to more effectively teach some of the concepts of gravity mentioned in Chapter 4. This instructional tool had to be usable within the existing lecture-laboratory-recitation framework, yet improve the quality of instruction. For this purpose, a set of worksheets was developed.

We hoped to create a worksheet that could be used both by students that had little or no background in physics and little understanding of the concept of force as well as those who had received instruction in the past on these subjects. Prior to its use in the Iowa State physics classes, the gravitation worksheets were field-tested in a community college introductory astronomy course for non-science majors. Since the astronomy students were largely of non-science backgrounds, three pages of introductory material (the first three pages of the Gravitation Worksheet in Appendix D) were included to give the students a working definition of force. Our anecdotal evidence is that these introductory pages were helpful in allowing students to complete the rest of the worksheet. The last of the three pages was included when administering the worksheets to the algebra-based physics students.

None of the three pages of introductory material was used with the calculus-based physics classes.

The remainder of the worksheet packet addresses some of the topics found to be problematic in the diagnostics. Material was included that addressed Newton's third law in the context of gravity, as well as the dependence of gravitational force on mass and on separation distance (parts of Newton's law of gravitation). The final page deals with gravitational force and the orbiting space shuttle.

The worksheets were tested in approximately one-fourth to one-third of each course's recitation sections (3 of 13 sections for the algebra-based course, 8 of 25 sections for the calculus-based course). The recitation sections ranged in size from 8 to 23 students. In both the algebra- and calculus-based physics classes, the worksheets were used during 30 minutes of one standard 50-minute recitation period. This left the regular recitation instructor 20 minutes to cover any other material he or she wished. Students in the "non-worksheet" sections had standard 50-minute recitations. Thus, there is no discrepancy of "time on task" between students in the two different samples. Students were asked to work in groups of three or four (though many of them worked in groups of two, due to physical room constraints.) The students were asked to start reading the worksheets and talk with each other in working through the questions. Two instructors, one of whom was the regular recitation instructor, walked around the room to make themselves available for students when they asked questions. The instructors periodically interrupted groups as well, asking them to explain what they had done.

Evaluation Of Worksheet Instruction

In one of the worksheet sections, the recitation instructor asked for the students' opinions of the value of the worksheets. The response was overwhelmingly positive; more than 90% of the comments were positive ones. One of the students wrote, "At first I thought that it was a bunch of b.s. But it really helped me a lot when I thought about what I was doing." Many students had similar sentiments, making it apparent that the students felt good about what they were doing.

However, determining if the students liked the instruction is different than determining if they learned more effectively from it than they may have in a standard recitation. While a positive student reaction is good news, it is worthwhile to employ another method to test the effectiveness of the worksheet instruction.

Final exam questions

As a measure of the effectiveness of the worksheet instruction, students in the Fall 1999 first-semester physics classes (both algebra- and calculus-based) at Iowa State were given two research-based questions on their final semester examinations. The research-based questions answered by the students were for credit, counting as a portion of their final exam grade. The final examinations were comprehensive in nature. A selection of several questions were submitted to the course lecturers, who, at their discretion, each selected two questions they deemed appropriate for their classes. One question (regarding Saturn's rings) was selected by both lecturers, while the other question was different for each class.

Calculus-based physics class results

The first final-exam question given to the students addresses gravitational forces acting on the Earth and an asteroid. The question reads as follows.

The rings of the planet Saturn are composed of millions of chunks of icy debris. Consider a chunk of ice in one of Saturn's rings. Which of the following statements is true?

*A) The gravitational force exerted by the chunk of ice on Saturn is **greater than** the gravitational force exerted by Saturn on the chunk of ice.*
*B) The gravitational force exerted by the chunk of ice on Saturn is **the same magnitude as** the gravitational force exerted by Saturn on the chunk of ice.*
*C) The gravitational force exerted by the chunk of ice on Saturn is **nonzero, and less than** the gravitational force exerted by Saturn on the chunk of ice.*
*D) The gravitational force exerted by the chunk of ice on Saturn is **zero**.*
E) Not enough information is given to answer this question.

This question is very similar to free-response #2, involving the forces between the Earth and an asteroid. The students improved overall from their first-semester performance on the similar free-response question (on which there had been 15% correct responses), with a noticeable difference between the students having worksheet instruction in recitation and those having standard recitation.

Table 5-1: Results, final exam question on Saturn's rings.

Response	Worksheet (N = 116)	Non-worksheet (N = 384)
A	3%	2%
B (correct)	**87%**	**61%**
C	9%	32%
D	1%	1%
E	1%	3%

$p < 0.0001$

We see a distinct difference in the results of this question when comparing the students in the worksheet instruction group with those in the standard recitations who did not do the worksheets. Using the two-sample test for binomial proportions to compare the worksheet and non-worksheet groups, we see that the difference in their correct answer rates is statistically significant with $p < 0.0001$.

The second final exam question read as follows.

Two lead spheres of mass M are separated by a distance r. They are isolated in space with no other masses nearby. The magnitude of the gravitational force experienced by each mass is F. Now one of the masses is doubled, and they are pushed farther apart to a separation of 2r. Then, the magnitude of the gravitational forces experienced by the masses are:

A) equal, and are equal to F.
B) equal, and are larger than F.
C) equal, and are smaller than F.
D) not equal, but one of them is larger than F.
E) not equal, and neither of them is larger than F.

The results shown in Table 5-2 show that once again the worksheet students answer the question on gravity with better results than the non-worksheet group, even though this question is more mathematical and not as explicitly conceptual (though a good conceptual understanding is helpful).

Table 5-2: Results, final exam question on lead spheres.

Response	Worksheet (N = 116)	Non-worksheet (N = 384)
A	16%	20%
B	5%	7%
C (correct)	**70%**	**45%**
D	4%	11%
E	5%	18%
$p < 0.0001$		

<u>Analysis of results</u>

The students using the worksheets in recitation do outperform the non-worksheet students on the final exam questions. How reliable is that result? First of all, we need to compare the student populations to see if there are significant differences between the two groups. One way to search for possible differences is to check their overall exam scores to see if one group scores significantly higher than the other. When checking the final exam scores for these two groups (not counting their scores on the two research-based questions cited above), we find that the exam average for the students in the worksheet sections is 53%, while the average is 48% for the non-worksheet group. The worksheet students, according to their final exam scores, tend to rank a bit higher in the class than the non-worksheet students, although the difference is not statistically significant according to the test for binomial proportions.

It is also useful to compare these students on their performance on non-gravity questions on the final exam. By generating a random number table, 10 questions from the Fall 1999 Physics 221 final exam were chosen. The following is a brief description of these "standard" questions.

Question	*Description*
132	*series-parallel circuit, determine current*
112	*block on inclined plane with friction, find force needed for movement*
114	*find potential between two point charges*
130	*capacitors in parallel, find total energy*
122	*cylinder rolling down inclined plane, find speed of center of mass*
113	*block on inclined plane with friction, find work done due to friction*
141	*definition of Kirchhoff's junction rule for circuits*
126	*find electric field in a capacitor*
111	*angular velocity of a mass on a string*
144	*inelastic collision, give conserved quantities*

Since the worksheet students perform about 5% better on the overall exam, we would expect that on the average, they would outperform the non-worksheet students by that margin. In Table 5-3, we see that to be the case for the standard test questions written by the course lecturer, but not for the gravity questions. While worksheet students do better on the standard test questions by an average of 4.4%, they do much better than the non-worksheet students on the gravity questions. The percentage of worksheet students answering the gravity questions correctly is higher than the percentage for the non-worksheet students by 25-26%.

Table 5-3. Physics 221 Fall 1999, final exam question results.

	Pct. Correct		
Question Number	Worksheet students	Non-worksheet students	Difference
"Standard" test questions			
132	46.6%	39.6%	7.0%
112	39.7%	33.6%	6.1%
114	44.0%	37.3%	6.6%
130	49.1%	48.8%	0.3%
122	20.7%	19.4%	1.3%
113	35.3%	31.5%	3.8%
141	35.3%	32.0%	3.3%
126	43.1%	35.9%	7.2%
111	59.5%	55.9%	3.6%
144	44.8%	39.6%	5.2%
Average			**4.4%**
95% confidence interval			2.7-6.1%
Gravity questions			
133 (lead spheres)	69.8%	44.8%	25.0%
145 (Saturn's rings)	87.1%	61.2%	25.9%
Average			**25.5%**
95% confidence interval			19.8-31.2%

Table 5-4: Adjusted student correct answer rates on final exam questions

Question	Worksheet (N = 116)	Non-worksheet (N = 384)	Adjusted p-value
Saturn’s rings	87%	65%	$p < 0.0001$
Lead spheres	70%	49%	$p < 0.0001$

The results of the analysis on the standard test questions suggest that our groups of worksheet and non-worksheet students are different by about 4% in terms of course performance. One way we can correct for this is to add 4% to the correct answer rates for the non-worksheet group. This results in a more conservative estimate of the difference in performance between the groups on the gravity exam questions. We note that the differences are still significant even with the correction included.

<u>Comparison with free-response data</u>

There are similarities between the two final exam questions above and questions 2 and 3 on the free-response diagnostic. Question 2 on the free-response diagnostic and the Saturn's rings question both rely on the student's understanding of Newton's third law in the context of gravity. Question 3 on the free-response diagnostic and the lead spheres questions both rely on an understanding of Newton's law of gravitation. The lead spheres question also relies on a correct application of Newton's third law, but that dependence can be removed if we only look at students who respond that the forces between the objects are equal (responses A, B, and C). By identifying the diagnostic and final-exam responses for a particular student, we can see if that student changed his or her understanding of the appropriate physical principles stated above.

A matched-set analysis of students in the worksheet and non-worksheet groups is useful for this comparison. Every student in the worksheet group is matched to a student in the non-worksheet group with the same exam score (N=106). Students who do not have a corresponding match or who have not answered both the diagnostic and exam questions are removed from the sample. The remaining matched sets of students then have the same overall exam average, removing that as a possible factor for one group outperforming the other.

In particular, we consider the percentage of students that answered the diagnostic question incorrectly but answered the final-exam question correctly. This is an indication that they have fixed their error and improved their understanding of that particular physical principle. Because we must leave out students who answered the diagnostic question correctly, our N values for the questions in Table 5-5 are less than N=106.

Table 5-5: Matched sets; percent of students fixing diagnostic error on final exam

	Worksheet	Non-worksheet	
FR#2 / Saturn's rings	84% (N = 82)	61% (N = 89)	$p = 0.0008$
FR#3 (A, B, and C only) / lead spheres	73% (N = 66)	65% (N = 63)	$p = 0.33$

This analysis suggests that the worksheet experience was significantly more effective than traditional instruction in helping students to fix their error in understanding Newton's third law. We also see that the percentage of students fixing their error with free-response question 3 is slightly higher for the worksheet group. However, this difference is not statistically significant (although it *was* significant for the full, un-matched sample). Without further study, it would be improper to claim that the worksheet experience had an effective impact on the students' understanding of Newton's law of gravitation.

Algebra-based physics class results

The final exam question about Saturn's rings on the algebra-based physics final is identical to the question on the calculus-based physics final (see above). The results show an improvement from the class' performance on a similar question with the Earth and an asteroid, free-response #2 (on which there had been 13% correct). There is a difference of 30% in the proportion of correct responses between the students in the worksheet and non-worksheet sections. In addition, there is a large improvement, up from 13% correct overall, from the class's response to the similar Earth-asteroid free-response #2 (see Table 4-11.) Both of these results are consistent with results from the calculus-based course.

Table 5-6: Physics 111 results from final-exam question on Saturn's rings.

Response	Worksheet Sections (N = 72)	Non-worksheet Sections (N = 211)
A	1%	0%
B (correct)	**71%**	**41%**
C	28%	52%
D	0%	4%
E	0%	3%
$p < 0.0001$		

A second question was asked on the algebra-based final exam that was a more mathematical question, though a good conceptual understanding of the problem would have been helpful. The question was similar but not identical to the question asked on the calculus-based final. It is slightly more complex, as three of the variables change instead of just two.

> *Two lead spheres of mass M are separated by a distance r; they are isolated in space with no other masses nearby. The magnitude of the gravitational force experienced by each mass is F. Now one of the masses is doubled, the other is tripled, and they are pushed farther apart to a separation of 3r. Then, the magnitude of the gravitational force on the larger mass is:*
>
> *A) F.*
> *B) 2F*
> *C) F/2*
> *D) 2F/3*
> *E) 3F/2*

Table 5-7: Physics 111 results from final exam question on lead spheres.

Response	Worksheet (N = 72)	Non-worksheet (N = 211)
A	14%	8%
B	13%	15%
C	7%	8%
D (correct)	**54%**	**51%**
E	13%	18%
$p = 0.66$		

There is a noticeable deviation from the calculus-based results on the lead spheres question. The difference between the worksheet and non-worksheet students that had been observed in the calculus-based course is not seen here. The difference in the percentages of students answering correctly is not statistically significant. It is difficult to give a specific reason for this, although the results suggest that the effect of the worksheet on the understanding of the algebra-based students was not the same as it was for the calculus-based students. Thus, one set of worksheets will not necessarily be equally effective in all levels of introductory physics courses.

Reliability of results

Unfortunately, records of attendance were not kept in the Physics 111 classes that received worksheet instruction. Separating the three recitation sections that did receive this instruction from the ten that did not does not isolate the worksheet students. Many students in the worksheet classes did not actually use the worksheets since not every student attended recitation on that day.

CHAPTER 6. CONCLUSIONS

Students studying physics at the college level appear to have misconceptions that are present in younger children as well. The diagnostic instruments used in this research proved useful in uncovering these misconceptions. Some of those misconceptions fade with increasing grade level and sophistication in science, while others do not. These misconceptions do not seem to be limited to non-science majors, but extend even to future biologists, engineers, and even physicists.

Research-based worksheet instruction shows potential benefits over traditional instruction, as evidenced by the performance of the algebra- and calculus-based students on final-exam questions dealing with gravity. The students using the worksheets performed better on the conceptual Saturn's rings question asked on the final exam. They also performed as well as (and in the case of the calculus-based class, better than) the rest of the class on the more mathematical lead-spheres question. In addition, the worksheets can be effectively incorporated into a traditional course without disrupting the flow of the course. While not necessarily appropriate for all classes, the worksheets appear particularly useful for calculus-based physics courses, and may have value in algebra-based physics courses as well.

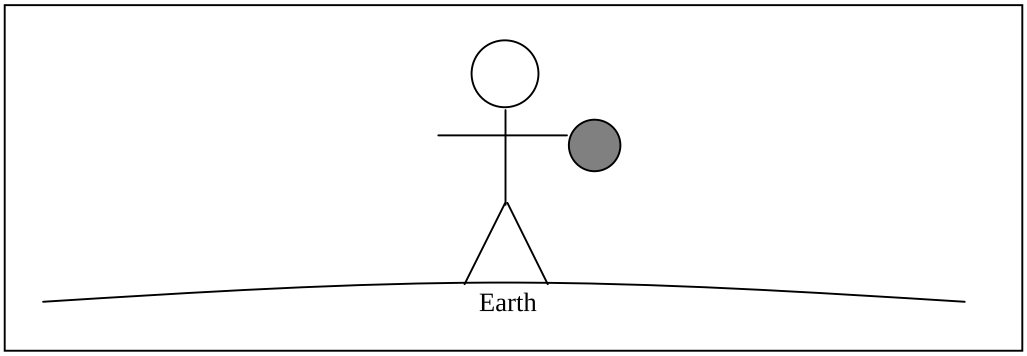
This work only scratches the surface; additional work needs to be done to develop a more complete understanding of students' concepts regarding gravitation, to investigate other facets of the learning of gravity, and to produce and test materials in an effort to improve instruction.

APPENDIX A: FREE-RESPONSE DIAGNOSTIC INSTRUMENT

NAME:_____________________________
CLASS:_____________________________

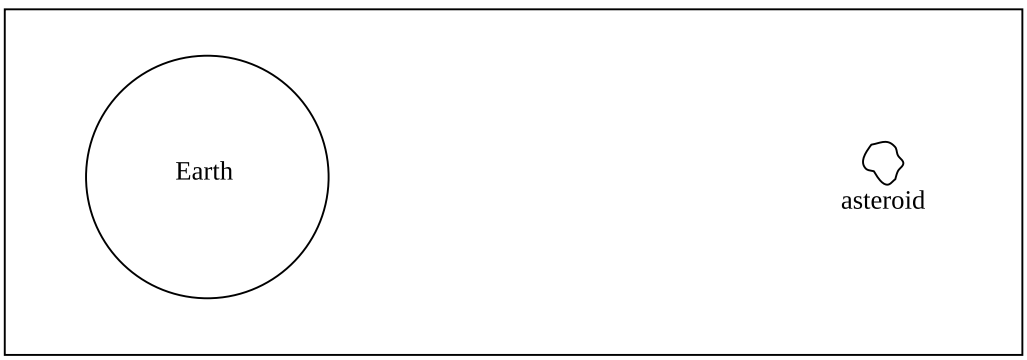
Gravitation Questions

1)



In the picture above, a person is standing on the Earth holding a ball in one hand. Draw the direction of the gravitational force exerted ***by*** the Earth ***on*** the ball in the picture above.

2)



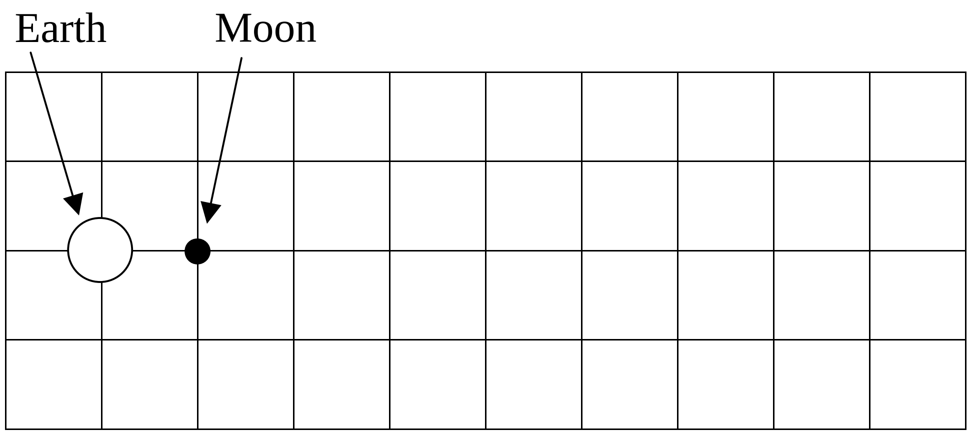
Refer to the picture above.

The magnitude of the force exerted ***by*** the asteroid ***on*** the Earth is [circle one]:

a) *larger than* the magnitude of the force exerted ***by*** the Earth ***on*** the asteroid.
b) *the same as* the magnitude of the force exerted ***by*** the Earth ***on*** the asteroid.
c) *smaller than* the magnitude of the force exerted ***by*** the Earth ***on*** the asteroid.
d) *zero.* (the asteroid exerts *no* force on the Earth)

Explain the reasoning for your choice.

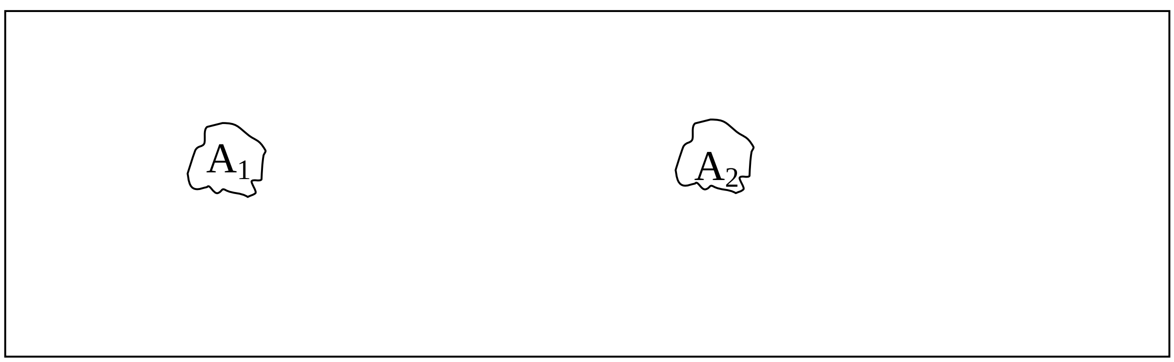
3)



The Moon experiences a gravitational force due to the Earth. Let's call the magnitude of this force $F_{\text{Earth on Moon}}$.

If you could pick up the moon and move it to another position, where would you put it to reduce the magnitude of $F_{\text{Earth on Moon}}$ to ¼ of its original value? Indicate the new position of the moon on the picture above and explain why you chose that position.

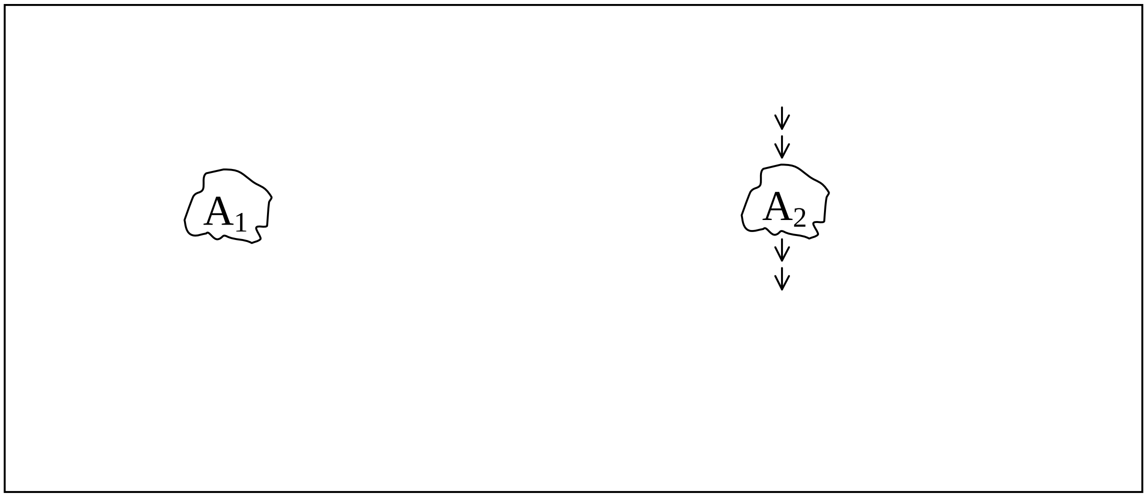
4)



The picture shown is a region of space showing two large, massive asteroids labeled A_1 and A_2. Both asteroids are free to move. **Assume nothing else is present**. Although the two asteroids are the same ***size***, asteroid A_1 is ***three times as massive*** as asteroid A_2. For each asteroid, draw one or more arrows on the asteroid to indicate the directions of any pushes or pulls which that asteroid experiences.

If two pushes or pulls are the ***same strength***, draw the arrows representing them ***the same length***. If one push or pull is ***stronger*** than another one is, draw a ***longer*** arrow to indicate the stronger push/pull. Draw a ***shorter*** arrow to indicate a ***weaker*** push/pull.

Explain the reason for your answer.

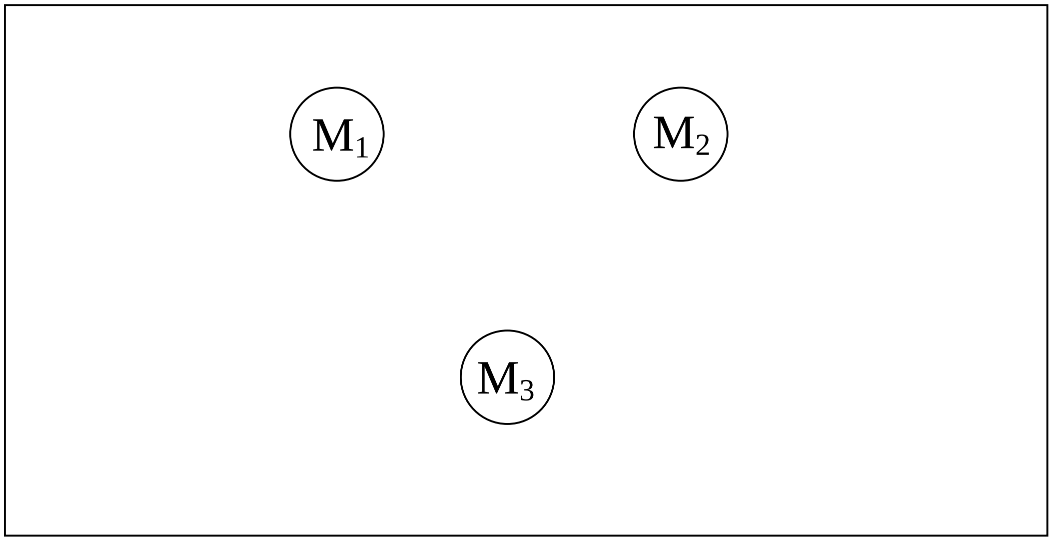
5) 

The picture above again shows a region of space showing two large, massive asteroids labeled A_1 and A_2. Both asteroids are free to move. **Assume nothing else is present.** Asteroid A_1 is still three times as massive as asteroid A_2. **However,** now asteroid A_2 is moving in the direction shown. As in Question #4, for each asteroid, draw one or more arrows on the asteroid to indicate the directions of any pushes or pulls that asteroid experiences.

If two pushes or pulls are the ***same strength***, draw the arrows representing them ***the same length***. If one push or pull is ***stronger*** than another one is, draw a ***longer*** arrow to indicate the stronger push/pull. Draw a ***shorter*** arrow to indicate a ***weaker*** push/pull.

Explain the reason for your answer.

6)

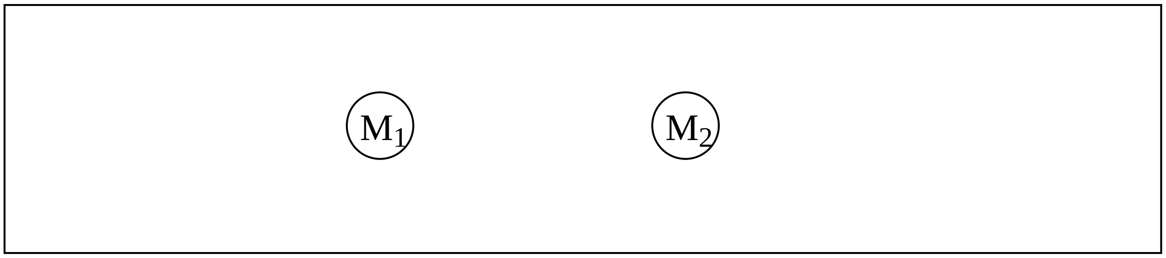


In the above diagram, three large moons are arranged so that they make an equilateral triangle. All three are the same size and have the same mass. Moons M_1 and M_2 are fixed in position and ***can not move.*** Moon M_3 is initially at rest, but is free to move.

Will moon M_3 move? [circle one] **YES** **NO**

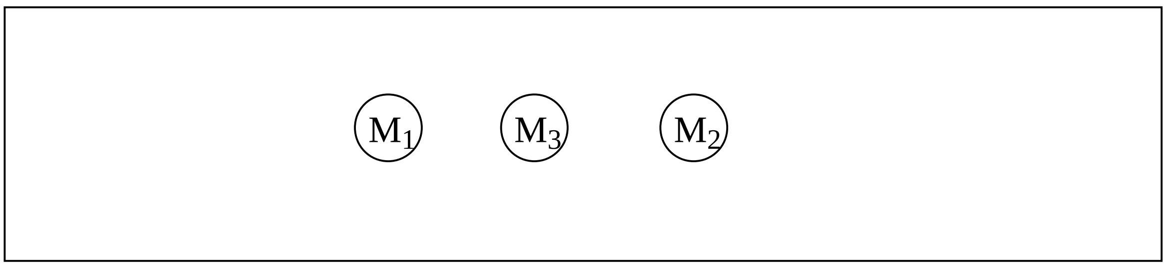
If yes, draw an arrow to indicate the direction that M_3 will move, and explain the reason for your answer. If no, explain why M_3 does not move.

7)



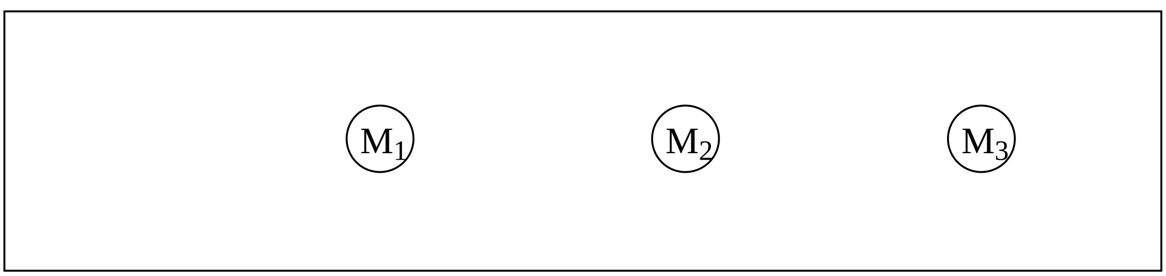
Two large masses M_1 and M_2 are in space as shown above.

A third mass M_3 is now placed in the position shown below.



A) [circle one] The net gravitational force on M_2 is now (**GREATER THAN,** **LESS THAN,** **THE SAME AS**) it was before M_3 was introduced. Explain your answer.

Now the mass M_3 is placed in a different position:



B) [circle one] The net gravitational force on M_2 is now (**GREATER THAN,** **LESS THAN,** **THE SAME AS**) it was before M_3 was introduced. Again, explain your answer.

8) Imagine that an astronaut is standing on the surface of the moon holding a pen in one hand.
A) If that astronaut lets go of the pen, what happens to the pen? Why?

B) After the astronaut lets go of the pen, what happens to this astronaut? Why?

9) A) Imagine that you are in a plane flying above the Earth.

i)	[circle one] Does the Earth exert a gravitational force on the plane?	**YES**	**NO**
ii)	[circle one] Does the Earth exert a gravitational force on you?	**YES**	**NO**

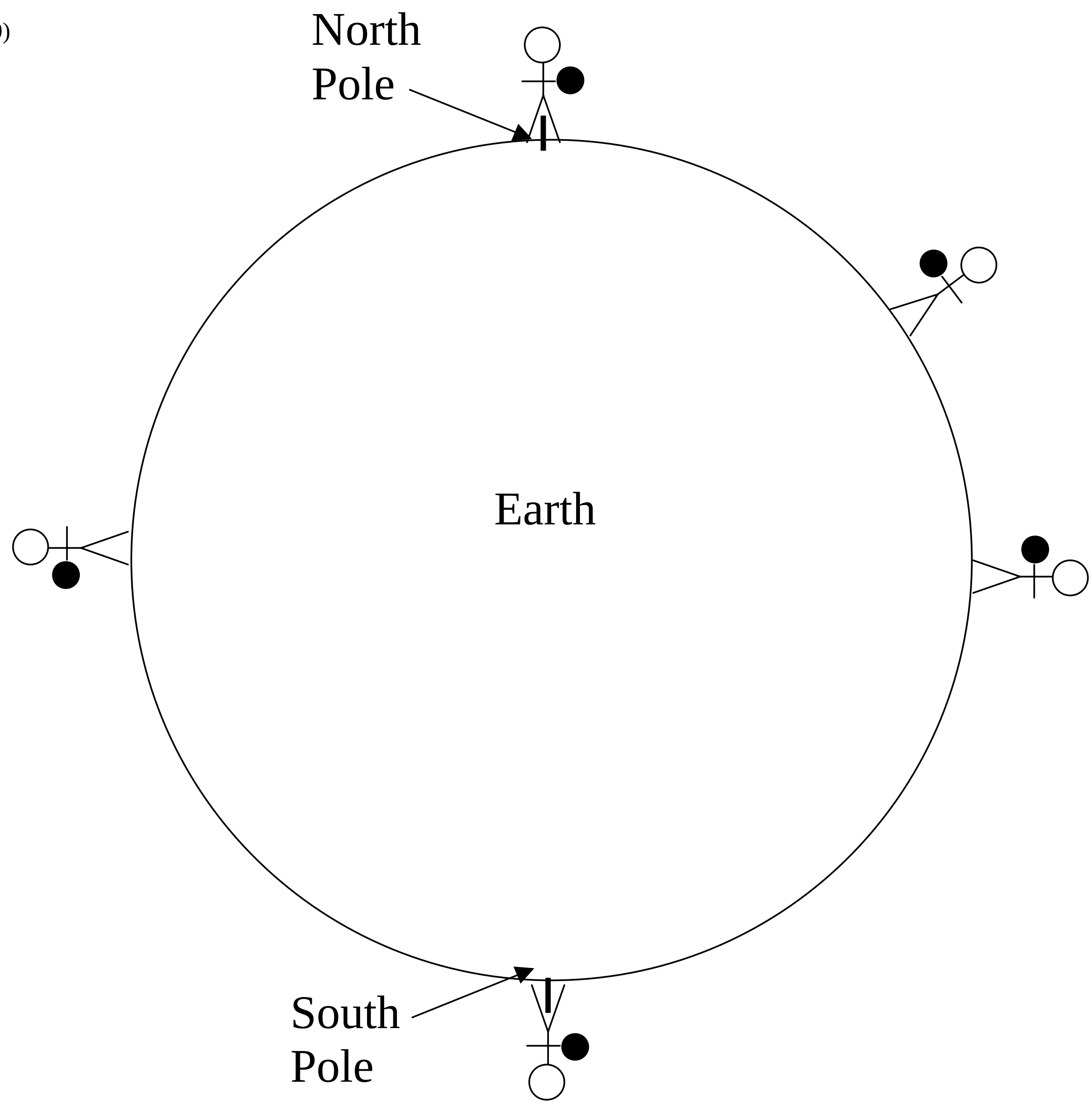
Explain why or why not for both i) and ii).

B) Now imagine that you are in the Space Shuttle orbiting the Earth.

i)	[circle one] Does the Earth exert a gravitational force on the Shuttle?	**YES**	**NO**
ii)	[circle one] Does the Earth exert a gravitational force on you?	**YES**	**NO**

Explain why or why not for both i) and ii).

10)



In the above picture, 5 people are standing in different places on the Earth. Each holds a black ball that he/she is about to drop. Draw an arrow for each ball showing which direction it will fall.

APPENDIX B: MULTIPLE CHOICE DIAGNOSTIC INSTRUMENT

Name_________________________

1. A 5-kg lead sphere is hanging 12 m from a 500-kg lead sphere. How does the gravitational force exerted by the 5-kg sphere <u>on the 500-kg sphere</u> compare with the magnitude of the gravitational force exerted by the 500-kg sphere <u>on the 5-kg sphere</u>? The forced exerted by the 5-kg sphere <u>on the 500-kg sphere</u> is:

   A. 100 times larger
   B. 10 times larger
   C. exactly the same
   D. 10 times smaller
   E. 100 times smaller

2. Suppose the distance between the spheres in #1 were somehow changed instantaneously to 36 m. What would happen to the magnitude of the gravitational force exerted by the 5-kg sphere on the 500-kg sphere?

   A. It would become 9 times larger than it was before.
   B. It would become 3 times larger than it was before
   C. It would be exactly the same as it was before.
   D. It would become one-third as large as it was before.
   E. It would become one-ninth as large as it was before.

3. Suppose the distance between the two spheres in #1 remained the same, but somehow the mass of the smaller sphere was changed to 20 kg. What would happen to the magnitude of the gravitational force exerted on the 500-kg sphere?

   A. It would become four times larger than it was before.
   B. It would become twice as large as it was before
   C. It would be exactly the same as it was before.
   D. It would become one-half as large as it was before.
   E. It would become one-fourth as large as it was before.

4. A. Which arrow points in the *direction* of the gravitational force exerted <u>by</u> mass #1 <u>on</u> the mass #3?

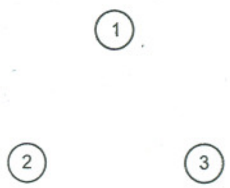
   A B C D E F G H

   B. Which arrow points in the *direction* of the gravitational force exerted <u>by</u> mass #2 <u>on</u> mass #1?

   A B C D E F G H

5. ***[2 points]*** In the diagram shown, three equal masses are shown. Draw and label (with the appropriate letter) <u>three</u> arrows, as follows:

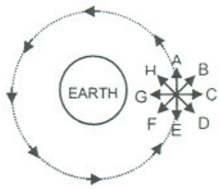
   A. the gravitational force of #2 <u>on</u> #1
   B. the gravitational force of #3 <u>on</u> #1
   C. the <u>net</u> gravitational force acting on #1



Make sure that the lengths of the arrows you draw are proportional to the magnitudes of the forces they represent!

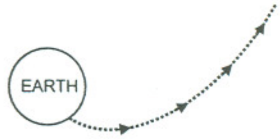
PLEASE TURN OVER → continued on other side

6. A satellite is orbiting the earth in a circular path as shown; the small arrows indicate its direction of motion. Which large arrow represents the gravitational force by the earth ***on*** the satellite when it is located at the position shown?



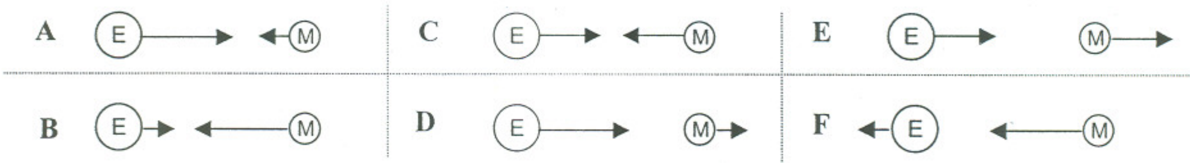
A B C D E F G H

7. A rocket is launched from earth at greater than the "escape velocity." This means that it continues on out into space, moving farther and farther away from the earth. Then the magnitude of the earth's gravitational force acting on the rocket:



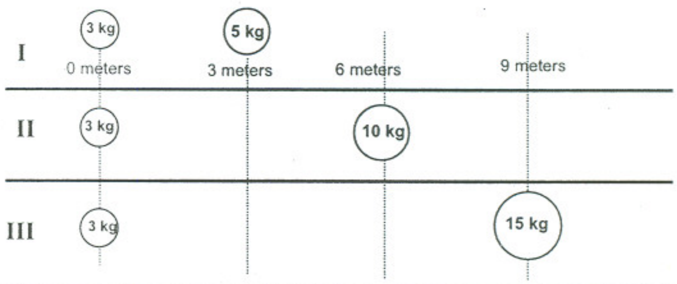
A. will eventually equal some constant value, greater than zero.
B. will eventually be exactly equal to zero, when the rocket reaches some particular distance from the earth.
C. will get smaller and smaller as the rocket gets farther away from the earth, but will never quite reach zero.

8. Which of these diagrams most closely represents the gravitational forces that the earth and moon exert ***on each other***? (Note: The mass of the earth is about 80 times larger than that of the moon.)



9. What can you say about the magnitude of the gravitational force on the 3 kg sphere in these three situations? The magnitude of the gravitational force on the 3 kg sphere is:

A. largest in I
B. largest in II
C. largest in III
D. equal in I and II, but larger than in III
E. equal in II and III, but larger than in I
F. equal in all three cases



10. In diagram #1, a large mass "M" is near to mass "m." In diagram #2, a smaller mass "M_2" has moved between the other two masses. What will happen to the magnitude of the ***net gravitational force*** acting on mass "m"?

A. It will increase, due to the force of the additional mass M_2.
B. It will stay exactly the same as it was in diagram #1.
C. It will decrease, because the mass M_2 shields some of the force originally coming from mass M.
D. It is not possible to say whether it will increase, decrease, or remain the same, with the given information.

11. Which arrow best represents the direction of the ***net gravitational force*** acting on mass #2 below?

(all masses are equal)

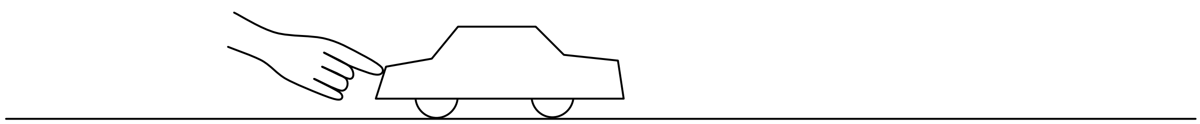
APPENDIX C. GRAVITATION WORKSHEETS

Name________________________

Gravitation worksheet

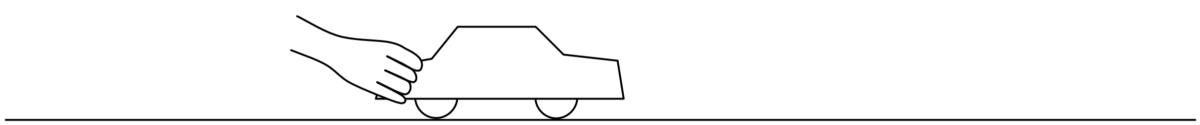
Recall Newton's 1st law of motion: An object at rest tends to remain at rest unless acted on by a **force**. But what exactly is a force? The simplest way to think of force is as a push or pull on an object. If an object is being pulled or pushed, then whatever is doing the pushing or pulling on the object is applying a force to that object.

1)



a) Imagine that you are pushing a toy car as shown above, which causes the car t to slowly start moving forward. Draw an arrow pointing in the direction you are pushing; label this arrow "**A**."

b) Suppose you now want to make the toy car speed up ***more quickly*** than in (a). Draw *another* arrow, near the first one, to represent this push. Label it "**B**." Draw the two arrows in (a) and (b) so that the ***longer*** arrow corresponds to the ***harder*** push.



Now imagine that you are gently pulling the car instead, as shown above. Your pull now causes the car to slowly start moving backward.

c) Draw an arrow pointing in the direction you are pulling; label this arrow "**C**."

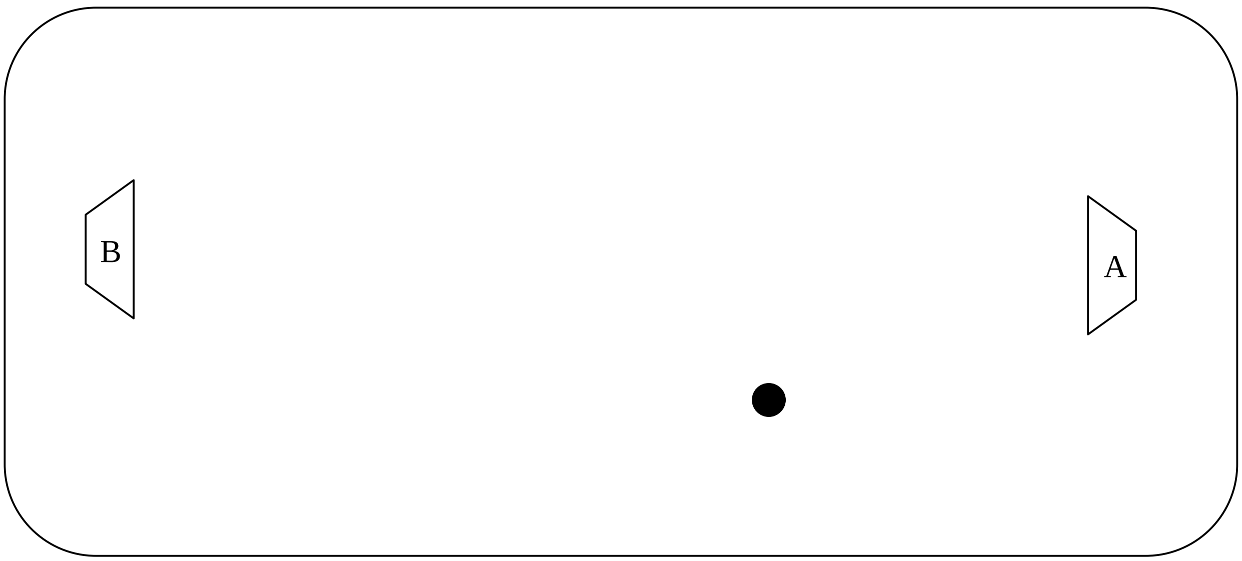
d) Suppose you now want to make the toy car speed up ***more quickly*** than in (c). Draw *another* arrow, near the first one, to represent this pull. Label it "**D**." Again, draw the two arrows in (c) and (d) so that the ***longer*** arrow corresponds to the ***harder*** pull.

These worksheets include some material reproduced with permission from *Workbook for Introductory Physics: Part II* (2000) by David E. Meltzer and Kandiah Manivannan.

You now have a scheme by which you can represent forces (pushes or pulls) in any situation! Forces can be represented by arrows because the arrows themselves represent the two important characteristics of a force:

i) First, the length of the arrow represents the strength or ***magnitude*** of the force. Longer arrows represent stronger forces.

ii) Second, the arrow must point in some ***direction***. The direction in which an arrow points indicates the direction in which the force is acting.



2) Above, a hockey puck sits motionless upon a flat, icy surface.

a) Draw an arrow on the puck indicating a force which will send it directly into net A (above right). Label this arrow "**A**."

b) Draw a second arrow on the puck indicating a force which will send it into net B (above left) even faster than force "**A**" sent it into net A. (Don't forget to think about the length of the arrow!) Label this new arrow "**B**."

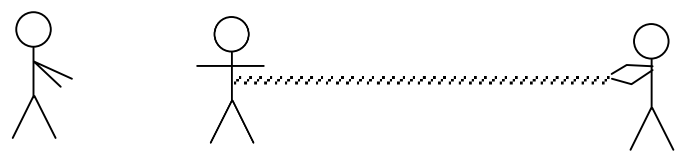
c) Draw an arrow on the puck indicating a force which will cause the puck to miss both nets. Label this arrow "**C**."

Name______________________

Gravitation worksheet

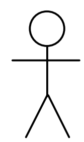
As you work through the rest of these worksheets, the phrase "force exerted ***by*** one object ***on*** another object" will appear often. Here's what that means. By "exerts,"it is meant that there is a force by one object on the other object. When we mention the force exerted ***by*** an object, that object is the source of the force which can act on other things. When we mention the force ***on*** another object, this object is the one experiencing the force.

3) Let's make a distinction between a "push" force and a "pull" force. Imagine that you are tied to one end of a rope, and someone else is <u>pulling</u> on the other end of the rope.



You

a) The person holding the rope is <u>pulling</u> you toward the right. Draw an arrow that represents the force exerted ***by*** that other person ***on*** you.

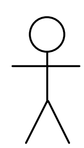


You

On the other hand, imagine that you are being <u>pushed</u> toward the right by a pole held by someone else.

You

b) The person holding the pole is <u>pushing</u> you toward the right. Draw an arrow that represents the force exerted ***by*** that person ***on*** you.



You

c) Are the forces exerted ***by*** the people in (a) and (b) ***on*** you in the same direction?
d) Would it be OK to draw the exact same picture for answers (a) and (b)? Explain.

Should you draw the same thing for (a) and (b)? YES, that's correct! Since pushes and pulls are both forces, and they both have the same effect, they can be represented by the same arrows. For clarity, you should always <u>place the tail of your arrow on the object</u> which is experiencing such a push or pull.

Name______________________

Gravitation worksheet
Please read all instructions!

As you work through these worksheets, the phrase "force exerted ***by*** one object ***on*** another object" will appear often. Here's what that means. When we mention the force "exerted ***by*** an object", that object is the source of the force which can act on other things. When we mention the force "***on*** another object", that object is the one experiencing the force.

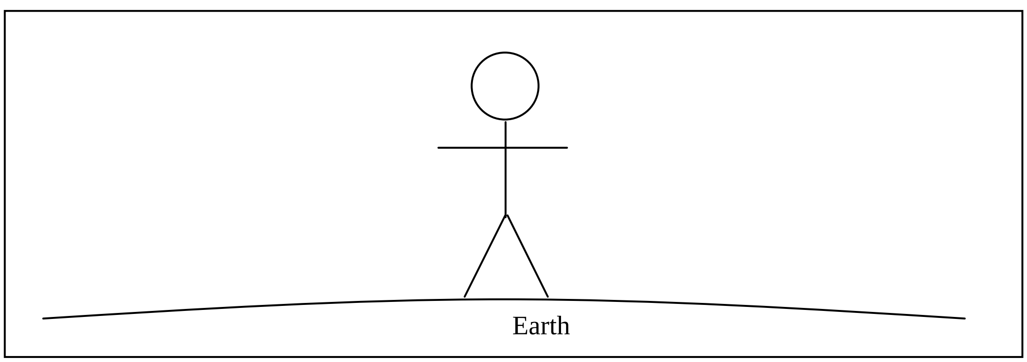
1) Newton's Law of Universal Gravitation is derived from Newton's 3 laws of motion. It describes the force on one massive object due to the presence of a second massive object. It is written as:

$$\mathbf{F = GM_1M_2/r^2}$$

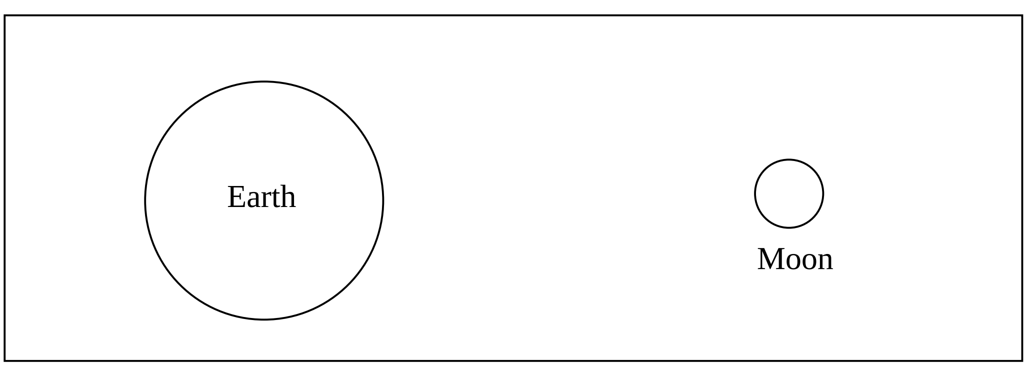
"M_1" and "M_2" represent the masses of each of the two objects, "r" represents the distance between the two objects, and "F" represents the magnitude (or strength) of the force which one of these massive objects exerts on the other. "G" is a constant (a number which can be looked up if you need a numerical answer) which is always the same, regardless of which two objects are being considered. Note that the force of gravity is a purely attractive force – that means that gravity is always a "pulling" force and never a "pushing" force.

Now let us move into more astronomical settings.

a) In the picture below, a person is standing on the surface of the Earth. Draw an arrow to represent the ***gravitational*** force exerted ***by*** the Earth ***on*** the person.



b) In the picture below, both the Earth and the Moon are shown. Draw an arrow to represent the force exerted ***by*** the Earth ***on*** the Moon. Label this arrow **(b)**.



c) Now, in the same picture (above), draw an arrow which represents the force exerted ***by*** the Moon ***on*** the Earth. Label this arrow **(c)**. Remember to draw the arrow with the correct length and direction as compared to the arrow you drew in **(b)**.

d) Are arrows (b) and (c) the same size? Explain why or why not.

e) Consider the magnitude of the gravitational force in (b). Write down an algebraic expression for the strength of the force. (Refer to Newton's Universal Law of Gravitation at the top of the previous page.) Use M_e for the mass of the Earth and M_m for the mass of the Moon.

f) Consider the magnitude of the gravitational force in (c). Write down an algebraic expression for the strength of the force. (Again, refer to Newton's Universal Law of Gravitation at the top of the previous page.) Use M_e for the mass of the Earth and M_m for the mass of the Moon.

g) Look at your answers for (e) and (f). Are they the same?

h) Check your answers to (b) and (c) to see if they are consistent with (e) and (f). If necessary, make changes to the arrows in (b) and (c).

i) (Circle one) When two objects of unequal mass (like the Earth and the Moon) exert gravitational forces on one another, the magnitude of the force exerted ***by*** the more massive object ***on*** the less massive object is [**SMALLER THAN, THE SAME AS, LARGER THAN**] the magnitude of the force exerted ***by*** the less massive object ***on*** the more massive object.

2) In the following diagrams, draw arrows representing force vectors, such that the length of the arrow is proportional to the magnitude of the force it represents. ***Use the same scale*** for all your exercises on this page and the next page. For example, if a force in diagram (i) has twice the magnitude of a force in diagram (ii), the arrows representing these forces will also have a length ratio of two to one.

Diagram (i): In this figure, two equal spherical masses (mass = “M”) are shown. Draw the vectors representing the gravitational forces the masses exert on each other. Draw your *shortest* vector to have a length equal to *one* of the grid squares.

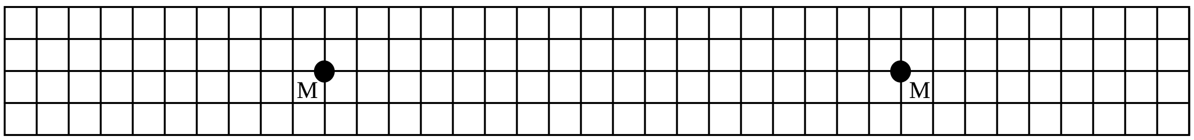


Diagram (ii): Now, one of the spheres is replaced with a sphere of mass 2M. Draw a new set of vectors representing the mutual gravitational forces in this case.

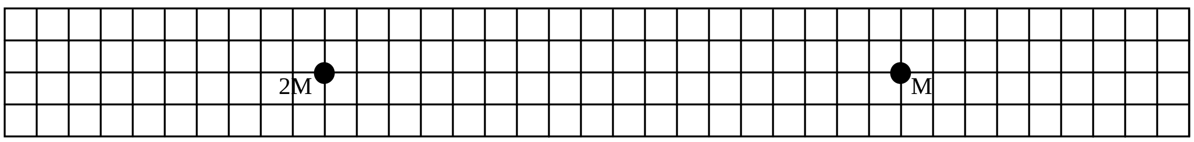
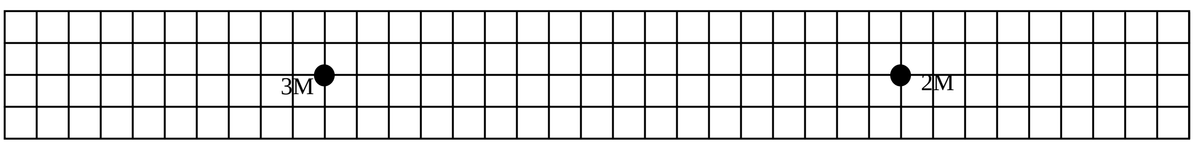


Diagram (iii): In this case, the spheres have masses 2M and 3M. Again, draw the vectors representing the mutual gravitational forces.



a) (Circle one) When the mass of an object is doubled, the magnitude of the gravitational force exerted ***by*** that mass ***on*** the other mass **[INCREASES, REMAINS THE SAME, DECREASES].**

b) (Circle one) When the mass of an object is doubled, the magnitude of the gravitational force exerted ***by*** the “unchanged” mass ***on*** the other mass **[INCREASES, REMAINS THE SAME, DECREASES].**

c) (Circle one) When two objects of unequal mass (like the Earth and the Moon) exert gravitational forces on one another, the magnitude of the force exerted ***by*** the more massive object ***on*** the less massive object is [**SMALLER THAN, THE SAME AS, LARGER THAN**] the magnitude of the force exerted ***by*** the less massive object ***on*** the more massive object.

d) Do your answers for (a)-(c) agree with what you drew in Diagrams (i)-(iii)? Discuss this with the students next to you.

e) Are your answers for (a)-(c) consistent with the Newton's Universal Law of Gravitation on the first page? If not, why not?

2) (continued) Once again, in the following diagrams, draw vectors representing the gravitational forces the masses exert on each other. Draw your arrows such that the length of the arrow is proportional to the magnitude of the force it represents. Continue to use the ***same scale*** that you used on the previous page in Diagrams (i)-(iii).

Diagram (iv): In this figure, the spheres have equal mass (mass = "M"), but their separation is half of what it was in Diagram (i).

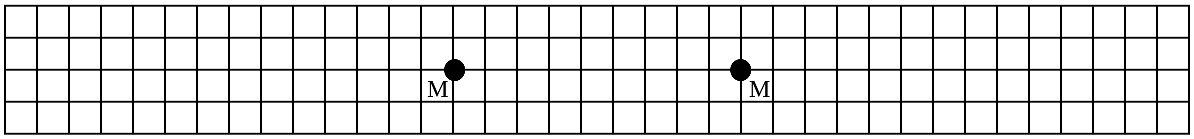


Diagram (v): Here the spheres have masses of M and 0.5M respectively, but again their separation distance is half of what it was in Diagram (i).

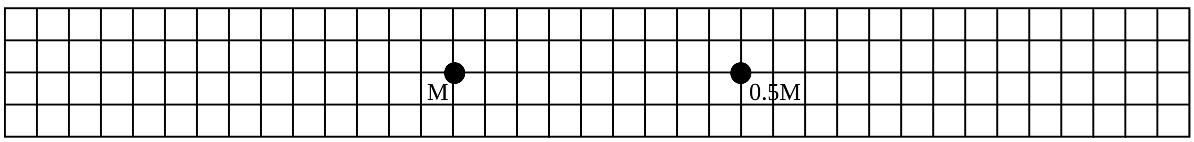
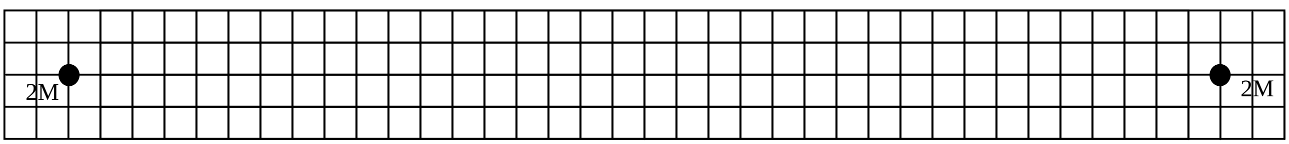


Diagram (vi): Here the spheres have equal mass 2M, but their separation is twice what it was in Diagram (i).



f) (Circle one) When the separation distance between two masses is *halved*, the magnitude of the gravitational force exerted ***by*** each mass ***on*** the other mass is **[4 TIMES LARGER, 2 TIMES LARGER, THE SAME, 1/2 AS LARGE, 1/4 AS LARGE]**

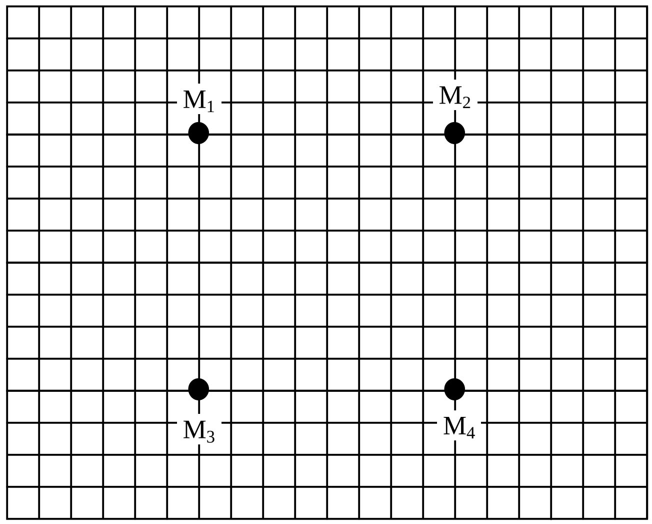
g) (Circle one) When the separation distance between two masses is *doubled*, the magnitude of the gravitational force exerted ***by*** each mass ***on*** the other mass is **[4 TIMES LARGER, 2 TIMES LARGER, THE SAME, 1/2 AS LARGE, 1/4 AS LARGE]**

h) (Circle one) When two objects of **unequal** mass (like the Earth and the Moon) exert gravitational forces on one another, the magnitude of the force exerted ***by*** the more massive object ***on*** the less massive object is [**SMALLER THAN, THE SAME AS, LARGER THAN**] the magnitude of the force exerted ***by*** the less massive object ***on*** the more massive object.

i) Do your answers for (f)-(h) agree with what you drew in Diagrams (iv)-(vi)? Discuss this with the students next to you.

j) Are your answers for (f)-(h) consistent with the Newton's Universal Law of Gravitation on the first page? If not, why not?

3) In this figure, four equal masses are arranged at the corners of a square. On each mass, draw arrows representing the gravitational forces exerted by the *other three masses* (so you will have three arrows attached to each mass). Make the lengths of the arrows proportional to the magnitudes of the forces. (Hint: pay attention to the arrow length on forces between two masses at opposite ends of the square.)



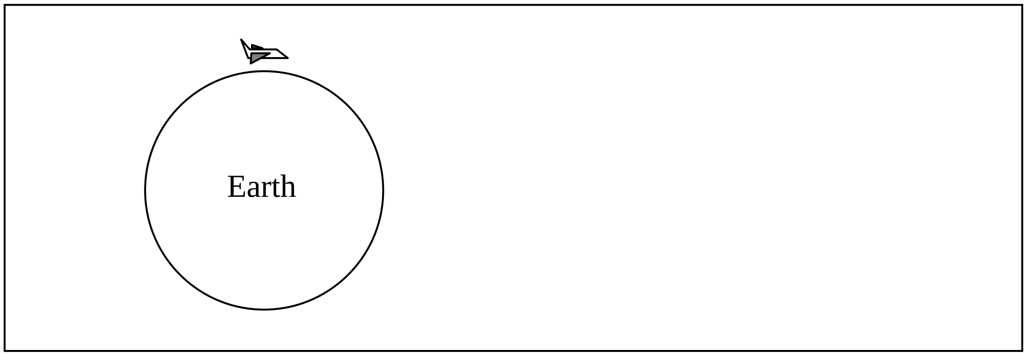
a) (Circle one) When two masses are moved farther apart, the magnitude of the gravitational forces between them [**INCREASES, REMAINS THE SAME, DECREASES**].

b) Are your arrows above consistent with your answer to (a)? Are they consistent with Newton's Universal Law of Gravitation (page 1)?

c) By what factor is the force exerted by M_4 on M_1 larger than (or smaller than) the force exerted by M_3 on M_1? Show your work.

d) If you are allowed to change the masses and locations of the objects in the above diagram, how could you make the force exerted by M_3 on M_1 4 times larger? (come up with at least 3 different ways)

4) a) Does the Space Shuttle have mass? Explain your answer briefly. (really!)



b) A picture of the Space Shuttle orbiting the Earth is shown above. Does the Earth exert a gravitational force on the Space Shuttle? If so, draw the appropriate arrow to represent that force. If not, explain why not.

c) Write down an algebraic expression for the gravitational force exerted by the Earth on the Space Shuttle. Use M_e for the mass of Earth and M_{ss} for the mass of the space shuttle. Define any other symbols you use.

d) Now consider an astronaut inside the space shuttle. Is the Earth exerting a gravitational force on the astronaut?

e) (Circle one) Compared to the gravitational force exerted by the Earth on the astronaut when standing on the Earth, the gravitational force the astronaut experiences when inside the orbiting Shuttle is **[ZERO, MUCH WEAKER, A LITTLE WEAKER, THE SAME, A LITTLE STRONGER, A LOT STRONGER]**

f) Calculate the ratio of the gravitational force on the astronaut *when in the shuttle* to the gravitational force on the astronaut *when standing on Earth*. The radius of the Earth is about 6400km, and during a normal mission the shuttle orbits about 500km above the Earth. (The altitude varies depending on the mission, but is not drastically different from this.)

g) If there is a gravitational force acting on the astronaut, how is it possible for the astronaut to feel weightless? (Hint #1: consider why you feel heavy right now.) (Hint #2: Draw free-body diagrams of the astronaut on the Earth and in the orbiting shuttle.)

APPENDIX D. SELECTED INTERVIEW TRANSCRIPTS

Interview 221-001
Regarding free-response #4:

S: This is… this is the deal. First it's the mass time acceleration, right? Okay. This is going around it. [referring to asteroid orbiting Earth, described with pencil in an elliptical circuit] It's got a bigger- it's got an acceleration which is bigger directed toward the Earth. Right?

I: Okay.

S: This here is stationary. [uses pencil to point to Earth] Well. There's no acceleration. But gravity is gravity. Aah, I'm just going to forget about the mass and I'm just gonna say that they've got the same force.

I: They feel the same force?

S: Yeah.

I: So what's your basis for saying that the forces are the same?

S: The basis for saying that is cause whatever pull you get [points to Earth] from here, this guy give you the same pull as that [points then to asteroid]- (the) opposite reaction to it.

Regarding the force between the Earth and the moon:

I: You said that there's a force on the moon in its normal position. And there's a force on this little chunk in its normal position.

S: [nods]

I: How do those two forces- the Earth pulling on the moon and the Earth pulling on the little chunk - how do those compare?

S: I'd say they're the same.

I: Same direction, same magnitude?

S: Yeah.

Referring back to free-response #4:

I: Describe the gravitational forces between [the two asteroids of differing mass].

S: Hmmm. As soon as I answer this [uses pen to point at #4] that disqualifies what my theory was for that [points at drawing of Earth and asteroid, #2]. I say that they have the same gravitational force between them…So the mass doesn't really matter. Wow.

I: Okay now you said this disqualified something. What did you mean by that?

S: I start thinking this [mass] just being three times this here. [points to drawing on worksheet of two different asteroids] This would have a bigger … pull. But then I was thinking how it don't really matter; how this here [points with pen toward asteroid on the separate Earth-asteroid sketch] I think will be the same pull even though this has a bigger mass. So now I have to take that this has the same gravitational force between them [now pointing and referring to the two asteroids in #4].

I: Okay.

S: Now when I go, I gotta see if mass makes some difference on gravity.

Regarding a modification to #4, cutting one of the asteroids in half:

I: You said that there was a force acting on this asteroid before I cut it in half. Is there a force acting on the piece left over after it's cut in half?

S: Yes.

I: How does the force acting on this compare to the force when the whole thing was there?

S: The same.

Regarding free-response #8a:

S: Okay. [The pen] would drop.

I: Why?

S: It would drop. Why? … Well the moon have a bigger force of gravity than the pen? [examines pen in hand] Well obviously they [pointing to Earth-asteroid diagram for #2] saying that they got the same force. Hmmm. Now I gotta rethink my gravity.

I: Okay.

S: It would go to the moon because the moon got a bigger mass. And I said here [points to sketched diagram of #4, asteroids of unequal mass] that it don't matter because of the mass. So I'm confused…

S: It will fall, I guess. Now I'm starting to think that the mass makes a difference.

I: What do you mean by the mass makes a difference?

S: Because the bigger the mass, since force is mass times acceleration, the bigger your force.

Regarding the space shuttle:

I: There's two questions there. Say you you've got some people in the space shuttle; the space shuttle's going around the Earth. First of all, does the Earth exert a gravitational force on the space shuttle?

S: Yeah.

I: Okay. Now…does the Earth exert a gravitational force on the people inside the space shuttle?

S: Don't it depend on how far you are from the surface of the Earth? Because I think after some- after you pass the atmosphere and stuff you just float. In other words the Earth is not exerting gravity on you. So how far are we talking?

I: A typical [shuttle] flight is 500 kilometers above the Earth.

S: After a certain height…you and the space shuttle and the people inside, the Earth don't exert no force on you. How would you float if you had gravity on you?

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